A Combinatorial Auction with Multiple Winners for Universal Service

Frank Kelly • Richard Steinberg
University of Cambridge, Statistical Laboratory, Cambridge, England CB2 1SB
University of Cambridge, The Judge Institute, Cambridge, England CB2 1AG

We describe a discrete-time auction procedure called PAUSE (Progressive Adaptive User Selection Environment) for use in assigning COLR (Carrier of Last Resort) responsibility for universal service. The auction incorporates synergies by permitting all combinatorial bids, is transparent to the bidders, allows for multiple winners, and minimizes the possibility of bidder collusion. The procedure is computationally tractable for the auctioneer and thus is very efficient to run. The inherent computational complexity of combinatorial bidding cannot be eliminated. However, in this auction the computational burden of evaluating synergies rests with the bidders claiming those synergies, while the auctioneer simply checks that a bid is valid.

(*Auctions; Combinatorial Bidding; Universal Service; Carrier of Last Resort; Telecommunications; Synergies*)

1. Introduction

It is not difficult to think of examples of auctions in which the value of a property to a bidder is increased if another property or group of properties is won by that bidder; this superadditive or synergistic effect may be bidder-specific. Auction authorities clearly have an incentive to structure their auctions so as to allow bidders to realize their synergies on combinations of properties in such a way that will be both fair to the bidders and practical to implement. The most obvious approach is to permit bids on groups of properties, called *combinatorial bids*.

Without the allowance of combinatorial bids, bidders will face "exposure risk" (Rothkopf, Pekeč and Harstad 1998). Suppose that an individual bidder has a synergy that is specific to him on a particular block of properties. He may find that an unsuccessful attempt to acquire the block leads him to commit to a price for a group of properties that is higher than what they are worth to him. Alternatively, he may be unwilling to risk bidding above the sum of his individual valuations, and thus may not be able to obtain the block for which the synergy makes him the efficient recipient.

Given the considerable importance of combinatorial bids, it may be surprising that few auctions—in either theory or practice—have allowed combinatorial bidding. This is undoubtedly due to the fact that combinatorial bidding is computationally burdensome. For example, Rassenti, Smith and Bulfin (1982) developed a combinatorial auction procedure for airport time slots to competing airlines. (See also Grether, Isaac and Plott (1989) and McCabe, Rassenti and Smith (1991).) Their optimization procedure is based on a mathematical programming formulation; however, the authors themselves point out that even a four-city problem will dictate a problem of enormous dimensions. In general, the auctioneer's problem of determining an optimal set of bids in a combinatorial auction is an NP-complete problem.

1.1 The PCS Auction

In the mid 1980s, the Office of Plans and Policy (OPP) of the Federal Communications Commission (FCC) issued a working paper that was soon to have an enormous impact. The paper, by Evan Kwerel and Alex D. Felker, proposed that FCC licensees be selected not by lottery or through hearings—as had been the standard practice to that point—but rather via auction (Kwerel and Felker 1985). This proposal came to fruition with the passage of the Omnibus Budget Reconciliation Act (1993), whereby Congress authorized the use of auctions for assigning the electromagnetic spectrum for Personal Communications Services (PCS).[1] The OPP was given the responsibility for managing the process, which included developing general spectrum auction rules.

The structure adopted was that of a *simultaneous multiple-round auction*: collections of licenses are auctioned simultaneously during discrete rounds, where no sale takes place until the bidding is concluded on all licenses. The PCS auction proceeds in three stages, each with an unspecified number of bidding rounds. At the end of each round, the highest bid becomes the leading bid, and the results are made available to all bidders before the start of the next round. At the end of the last round of the third stage, the leading bidder on each property is designated the sole winner on that property. Detailed rules of the auction can be found in FCC (1993).[2] Key aspects include:

Activity Rules. *Eligibility Requirements.* Each round, a bidder is designated *active* on a particular property if either he has the leading bid from the previous round or has submitted an acceptable improving bid in the current round. In Stage i, bidders are required to remain active on licenses covering a population which is at least A_i percent of the total population for which they wish to remain eligible to bid, where $A_3 > A_2 > A_1$. These eligibility requirements are intended to thwart the "deception effect," whereby a firm might bid cautiously, waiting to see how the others bid while not revealing its own interests until late in the auction.

Bid Increments. In order to be acceptable, a bid must improve the previous leading bid by at least the specified minimum amount (e.g., 5% of the leading bid) set by the auction authority. This helps maintain the speed of the auction.

Transition Between Stages. The auction moves from the first stage to the second when there are no bids on more than T_1 percent of the population base for three consecutive rounds, and from the second stage to the third when there are bids on no more than T_2 percent of the population base (e.g., values of 5% have been used). The auction closes when bidding stops on every license.

Bid Waivers. The activity rules are balanced by an allocation of a small number of bid waivers to each player to be used at will to maintain eligibility for a round without meeting the eligibility criteria. Bid waivers may be viewed as an effort to increase bidder flexibility.

---

[1] Personal Communications Services are a broad family of mobile communications services that allow people access to the Public Switched Telephone Network (PSTN) regardless of where they are located. PCS includes not only POTS ("Plain Old Telephone Service") but also data, facsimile, video communication, and other services.

[2] The PCS auction was developed through the efforts of many people. Rothkopf, Pekeč and Harstad (1998) provide complete details.

Bid Withdrawals. A leading bidder is permitted to withdraw his bid during the course of the auction, but is penalized by being required to pay the difference between his bid and the price for which the license is ultimately sold; a winning bidder withdrawing after the close of the auction suffers an extra penalty. Bid withdrawals may be viewed as an effort to reduce the exposure risk to bidders attempting to realize their synergies. The FCC has not yet permitted combinatorial bids in the spectrum auctions. Rothkopf, Pekeč and Harstad (1998) persuasively argue that the disallowal of combinatorial bids is a consequence of the economists' briefs—in particular that of McAfee (1993)—that argued that the only choice was between completely disallowing, or permitting all possible, combinatorial bids, and the latter option in the worst-case scenario would be computationally intractable.

1.2 Universal Service and Carrier of Last Resort

Combinatorial auction design is a live issue with regard to the Telecommunications Act of 1996, which includes a requirement that a joint board of Federal and state regulators consider ways to reform the current methods of providing universal service subsidies for high-cost areas (Telecommunications Act 1996).[3] Several parties have advocated that competitive bidding be used to determine these subsidies. A policy paper issued by Citizens for a Sound Economy Foundation (Leighton 1996) states the case clearly:

> Various methods have been discussed, but the best way may be to allow the marketplace—through competitive bidding—to determine the size and scope of any subsidies. Appropriately designed auctions may serve as an effective means to minimize the costs of service in these areas, while providing equal or greater quality of service.

The paper also assumes that such an auction will allow for multiple winners:

> More than one bidder could win a subsidy in the auction, though rules would have to be set forth to clarify which bidders would be eligible for support. While no provider would be denied the right of entry, the subsidy might be limited to those who participated in the auction and offered a sufficiently low bid.

A U.S. telephone carrier that undertakes various obligations as a condition for receipt of universal service support is called a *Carrier of Last Resort* (COLR). In a COLR auction, firms would bid for subsidies on geographic areas, where the winning firms are those that have submitted the lowest subsidy bids. Synergies—in particular, cost synergies—may be a significant consideration in the design of a COLR auction; i.e., it is conceivable that a firm might find it less costly to provide COLR service for a particular geographic area if it serves it together with some collection of other areas.

1.3 Computationally Tractable Auctions

Attempts to make the combinatorial auction design problem tractable through specific restrictions on the bidding mechanism have taken the approach of considering specialized structures that are amenable to analysis. Rothkopf, Pekeč and Harstad (1998) discuss

---

[3] Universal service subsidies are government subsidies given to telecommunications service providers. These subsidies are intended to promote the availability of quality services at just, reasonable and affordable rates to all consumers, including those in high-cost areas (as well as those in low-income, rural and insular areas) at rates that are reasonably comparable to those charged in urban areas.

these approaches, and ask: What are the least restrictive structures that would result in a computationally tractable problem for the auctioneer to determine the revenue-maximizing outcome?[4] They consider several different structures and constructively show that those they have considered are indeed computationally tractable.

While this manner of dealing with the computational intractability violates what might be termed the "McAfee principle" of allowing *all if any* combinatorial bids, Rothkopf, Pekeč and Harstad show in a certain sense how far one can go in this direction: in each instance, they demonstrate their approach to be best possible by proving that the next level of generality will result in an NP-complete problem. For example, they show that allowing bids on arbitrary doubletons reduces to finding a maximal weighted matching in a graph. By the algorithm of Edmonds (1965), such a matching can be found in $O(n^3)$ time. However, they also show that allowing arbitrary tripletons reduces to the 3-set packing problem, which is NP-complete (Karp 1972).

Rothkopf, Pekeč and Harstad do make a convincing case that there are many natural ways to restrict the nature of the bids so as to make combinatorial auctions computationally tractable. However, they also warn that this then raises the problem of deciding *which* combinations of bids to allow.

1.4 Adaptive User Selection Mechanism (AUSM)

Thus far, the only auction approach that has allowed for all possible combinatorial bids is the Adaptive User Selection Mechanism (Banks, Ledyard and Porter 1989). In AUSM, bids can be submitted at any time, where bidders may incorporate any unwithdrawn, currently unsuccessful bids into their own bids, with bidding stopping according to some pre-specified stopping rule. (The original rule specified that the auction stops when no new bid is made soon enough after the last bid.) However, this procedure is susceptible to the "threshold problem." Specifically, a group of players who jointly desire a subset of properties may have difficulty coordinating their bids to displace a single bid on those properties. To address this problem, Banks, Ledyard and Porter designed a modification of AUSM that makes use of a "stand-by queue" in which bidders announce to all other bidders via a bulletin board their willingness to pay a certain price for a specific combination of licenses. As described in Bykowsky and Cull (1993): "In essence, the stand-by queue serves as a voluntary contribution mechanism in which perspective contributors attempt to move, in a repeated game context, to a mutually desirable equilibrium."

While a stand-by queue can help overcome the threshold problem, it is less effective at dealing with the related "free-rider problem." A group of players who jointly desire a subset of properties may, in principle, be able to coordinate their bidding via a bulletin board, but each player from the group will have an incentive to wait for the others in the group to improve their bids, thus retaining more of the benefit from the joint bid for itself. Cramton (1997) also points out that AUSM weakens a central advantage of auctions, viz., *transparency*. Transparency means that a losing bidder who offered a higher bid for part of a combination should always be able to see why he lost. Unfortunately, this is generally not the case under AUSM.

---

[4] Here "computationally tractable" is the standard concept that an upper bound on computation time for the given class of computational problems can be expressed as a polynomial function of the size of the input.

1.5 Jump Bidding

A potential problem with simultaneous, multiple-round auctions has been reported by McAfee and McMillan (1996). In the portion of the PCS auction known as the MTA auction,[5] aggressive bidding in early rounds took the form of "jump bidding": entering bids far above that required by the minimum bid increment. The intention of this tactic, which we call more specifically *price-jump bidding*, is to warn weaker rivals against competing on specific properties. A COLR auction would be a low-bid auction, and jump bidding would mean entering bids far *below* that required by the minimum bid increment. A combinatorial auction presents the possibility of another type of jump bidding, *block-jump bidding*, in which a bid by a powerful player for a block of several properties could be effective at preventing small players from piecing together a comparable composite bid, i.e., the threshold problem.

Although McAfee and McMillan describe jump bidding as merely a part of bidder strategies, clearly it is of concern to the FCC (1997, paragraph 143):

> Several commentators suggest that jump bidding is not a problem of serious concern. Some theoretical literature, however, suggests that bidders could use jump bidding to manipulate the auction process and potentially reduce efficiency of the auction.

1.6 Bidder Collusion

Experience with the PCS auction has revealed the possibility for a type of bidder collusion that we designate *property-preference signaling.* The idea is that bidders signal their strong interest to win specific properties in subsequent rounds by encoding this information in their bid prices, and then make use of this information to bid noncompetitively on each others' designated properties. This tactic can carry the additional benefit—like price-jump bidding—of warning away weaker rivals. As reported in *The Economist* (1997), this has been alleged to have occurred in a PCS auction held in January 1997. Presumably, what made the encoding of price information possible was the high order of precision to which bids could be submitted. According to *The Economist*, the FCC is considering in future auctions to continue to accept bid prices to a high order of precision–which tends to prevent against tie bids–but to report them to a lower order of precision. We will tacitly assume in the sequel that this feature is incorporated into our auction design.

A second form of collusive signaling, which we call *price-level signaling*, is conceivable in the situation where multiple winners are allowed. In this case, two or more players become aware that they are actively interested in the same property on which they have the potential to share as multiple winners. Consequently each firm bids noncompetitively so as to maximize its subsidy as a joint winner on that property.

1.7 Universal Service Auction Desiderata

Based on the preceding, we can present a list of properties that we would want to see in an auction design for universal service.[6] The auction should allow for all combinatorial

---

[5] The MTA auction ran from December 1994 to March 1995 and sold broadband licenses covering the 51 "Major Trading Areas," or MTAs, into which the United States is divided.

[6] Following Milgrom (1996a), we make the simplifying assumption that the fixed costs of service are the same across bidders.

bids and allow for multiple winners. The deception effect can be addressed in activity rules along the lines of those used in the PCS auction. In addition, the auction should:

(a) Be transparent to the bidders;
(b) Present the auctioneer with a tractable bid evaluation problem;
(c) Have a bounded completion time;
(d) Prevent against jump bidding and mitigate the threshold problem; and
(e) Minimize the opportunity for bidder collusion, specifically, price-level signaling.

2. The PAUSE Auction

We describe a discrete-time auction procedure called PAUSE (Progressive Adaptive User Selection Environment) for use in assigning COLR responsibility.[7] More specifically, PAUSE is a two-stage procedure, where:

> Stage 1 is a simultaneous, multiple-round auction, conducted in three substages, with progressive eligibility requirements and an improvement margin requirement, with bidders submitting bids on individual properties; and
>
> Stage 2 is a simultaneous, multiple-round auction, conducted in two substages, with progressive eligibility requirements and an improvement margin requirement, with *composite bids* to facilitate realization of player synergies.

The main contribution of this paper is the combinatorial auction described in Stage 2, and there are several consistent ways that Stage 1 might be organized. The structure of Stage 1 we describe below makes minimal changes from that used in the PCS auction. In Appendix A, we describe a structure for Stage 1 as a menu auction.

PAUSE is designed to be fully general in that every possible combinatorial bid is available to the bidders. If, however, the auctioneer wishes to restrict the bids in any manner that he finds convenient to verify, the auction structure will accommodate this, and the auctioneer can announce to the bidders a list of attributes a bid must have. (An example of such an attribute might be: "Bids that are combinatorial are to be composed of geographically contiguous subsets of the properties.") This is formalized below.

2.1 Definitions

Label *properties* $j \in J$, and *blocks* $k \in K$, where $K = K(J, A)$ is a set of subsets of J defined by a set of *attributes* A that are computationally tractable for the auctioneer to verify for each member of K. Let

$$K_n = \{k \in K(J, A) : 1 \leq |k| \leq n\},$$

where $|k|$ is the number of properties in block k. A *partition* $P = (p_1, p_2, \ldots, p_r)$ is a collection $p_1, p_2, \ldots, p_r \in K$ such that $\bigcup_{i=1}^{r} p_i = J$, and $p_i \cap p_j = \emptyset, i \neq j$. A *composite bid* comprises a partition $P = (p_1, p_2, \ldots, p_r)$, together with an *evaluation*:

$$(C(P); c(p_1), c(p_2), \ldots, c(p_r))$$

---

[7] Note that this auction structure can, with minor modifications, be adapted for use as a combinatorial spectrum auction.

where

$$C(P) = \sum_{i=1}^{r} c(p_i), \tag{1}$$

and $c(p_i)$ is the *bid* for block p_i.

To be more precise, $c(p_i)$ is the *value of the bid for block* p_i. A composite bid consists of $3r + 1$ pieces of information, capable of registration in a database. The first piece of information is the total value of the composite bid, $C(P)$. The $3r$ pieces of information are, for each i ($i = 1, 2, \ldots, r$): (1) the specification of the block p_i, (2) the value of the bid on the block $c(p_i)$, and (3) the identity of the bidder for block p_i. All $3r + 1$ pieces of information are available from the database to all bidders. Note that $c(p_i)$ is the *total subsidy for block* p_i. It corresponds to a *subsidy per subscriber in block* p_i of $c(p_i)/\|p_i\|$, where $\|p_i\|$ is the total number of subscribers[8] in all the properties in p_i.

2.2 Two Stages of the PAUSE Auction

Stage 1: Bidding on Individual Properties

The Bidders. Each bidder submits a collection of bids on individual properties. In each round there is an *improvement margin requirement*:

> The new bid must improve on the previous best bid on that property by *at least* ϵ and *strictly less than* 2ϵ.

The Auctioneer. In each round, for each property the auctioneer checks that a bid on that property is *valid* by checking:

> *Increment validity*: The bid satisfies the bounds of the improvement margin requirement.

In each round, the lowest valid bid on each property is accepted. The round ends when bidding ends on all properties. Stage 1 is divided into three substages. At the conclusion of the third substage, the leading (i.e., lowest) bids on the properties are registered to their respective owners, and the auctioneer announces the number of multiple winners that will be accepted and necessary for property j, as determined by the rule below.

Activity Rules. A bidder is designated *active* on a property if he has the leading bid from the previous round or submits an acceptable bid in the current round. Each of the three substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the three stages, 60 percent, 70 percent and 80 percent of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to 2 occurs when there are bids on no more than 10 percent of the subscribers for three consecutive rounds, from substage 2 to substage 3 when there are bids on no more than 5 percent of the subscribers for three consecutive rounds.

---

[8] In this document, by subscribers we mean subscribers counted under the Universal Service provisions for support for high cost areas.

Multiple Winners. At the conclusion of the third substage, the auctioneer announces the number of winners on each property as determined by the *outcome rule*:[9] (1) if at least one competing bid is within $M_1 = 15$ percent of the lowest bid, then all who bid within M_1 percent of the lowest bid are designated as winners; (2) if no competing bid is within M_1 percent but one is within $M_2 = 25$ percent, then the two lowest bidders are winners, and (3) if no bid is within M_2 percent, then there is a single winner, viz., the lowest bidder. The number of multiple winners on each property j at the end of Stage 1 is denoted by $m(j)$. Before the start of Stage 2, property j is replaced by $m(j)$ properties $j_1, j_2, j_3, \ldots, j_{m(j)}$, each allocated a nominal number of subscribers equal to $\mathrm{sub}(j)/m(j)$, where $sub(j)$ denotes the number of subscribers in property j.

Stage 2: Combinatorial Bidding

The Bidders. Each bidder submits a single composite bid (which, by definition, includes all the properties in the auction). In each round there is an *improvement margin requirement*:

> Let b be the number of *new* bids in the composite bid. The new evaluation must improve on the previous best evaluation by *at least* $b\epsilon$ and *strictly less than* $2b\epsilon$ (i.e., an average improvement per block of at least ϵ but less than 2ϵ).

Each bidder's partition $P = (p_1, p_2, \ldots, p_r)$ is restricted to $p_i \in K_n$, where $c(p_i)$ is either a new bid for block i, or a registered bid. Initially, $n = 2$. For a composite bid to be valid,[10] for each property j the bid must not allocate j_s and j_t $(s \neq t)$ to the same player. In this stage of the auction, the bidder identities are made public.[11] Thus the validity of a composite bid—and in particular the requirement that the bid does not allocate j_s and j_t to the same player—can be checked by the player constructing the composite bid.

The Auctioneer. In each round, the auctioneer checks that a composite bid is *valid* by checking:

(i) *Bid validity*: Each bid which is asserted to be registered in the database is indeed so registered; that new bids identify correctly the bidder for block p_i, and satisfy $p_i \in K_n$; and, for each property j, the composite bid does not allocate j_s and j_t $(s \neq t)$ to the same player.

(ii) *Evaluation validity*: Equation (1) holds, i.e., the value $C(P)$ of the composite bid is indeed the sum of the bids on each of its blocks, and

(iii) *Increment validity*: The bid evaluation $C(P)$ satisfies the bounds of the improvement margin requirement.

---

[9] This is the outcome rule proposed by Milgrom (1996a), who points out that other formulas for multiple winners are possible. The concept of the outcome rule itself is due to Milgrom.

[10] Note that synergies are accounted for via composite bids. Thus, to allow for multiple winners, players need to check the validity of their composite bids.

[11] McAfee and McMillan (1996) report that in the MTA broadband PCS auction the FCC revealed bidders' identities, judging the risk of collusion to be outweighed by the benefits of information. They point out: "Bidders' identities are useful to the bidders for evaluating the meaning of others' bids, reducing the winner's curse and generally assisting sensible bidding." They add that "it takes only one maverick bidder to upset an attempt at collusion," and provide an example from the MTA auction.

In each round of Stage 2, the new collection of bids on the blocks $\{c(p_i)\}$ are registered in the database to their respective owners, and the lowest valid composite bid is accepted. Thus, in each round, the auctioneer accepts *one* composite bid from among all the composite bids submitted by the players,[12] but registers all the valid composite bids. The round ends when bidding ends. Stage 2 is divided into two substages.

The size of the bid increment, ϵ, and the rate of increase of the block size limit, n, are used by the auctioneer to control the speed of the auction, in conjunction with the activity rules. (Figure 1 provides a simple illustration of combinatorial bidding.)

Activity Rules. A bidder is *active* on a property if his bid on a block containing that property forms part of the accepted composite bid of the previous round, or if he submits a valid bid in the current round on a block containing that property. Each of the two substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the two stages, 90 percent and 98 percent of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to substage 2 occurs when there are bids on no more than 10 percent of the subscribers for three consecutive rounds.

Multiple Winners. At the conclusion of Stage 2, the $m(j)$ winners on property j are each designated a $1/m(j)$ share of the responsibility on property j. Specifically, the contractual obligation carried by each player is as follows:

(i) *The player will receive his bid subsidy per subscriber on* up to $1/m(j)$ *of the total number of subscribers in that property*;

(ii) *The regulatory authority may require any winner in that property who is not serving the full amount of his contractual share to serve any unserved subscriber in that property.*

The particular subscribers that constitute a contractual share are not specified; the player will compete for these subscribers with the other winners on that property. There is thus an incentive for players to actively seek to serve their share of subscribers, lest they be required to serve subscribers not of their choice.

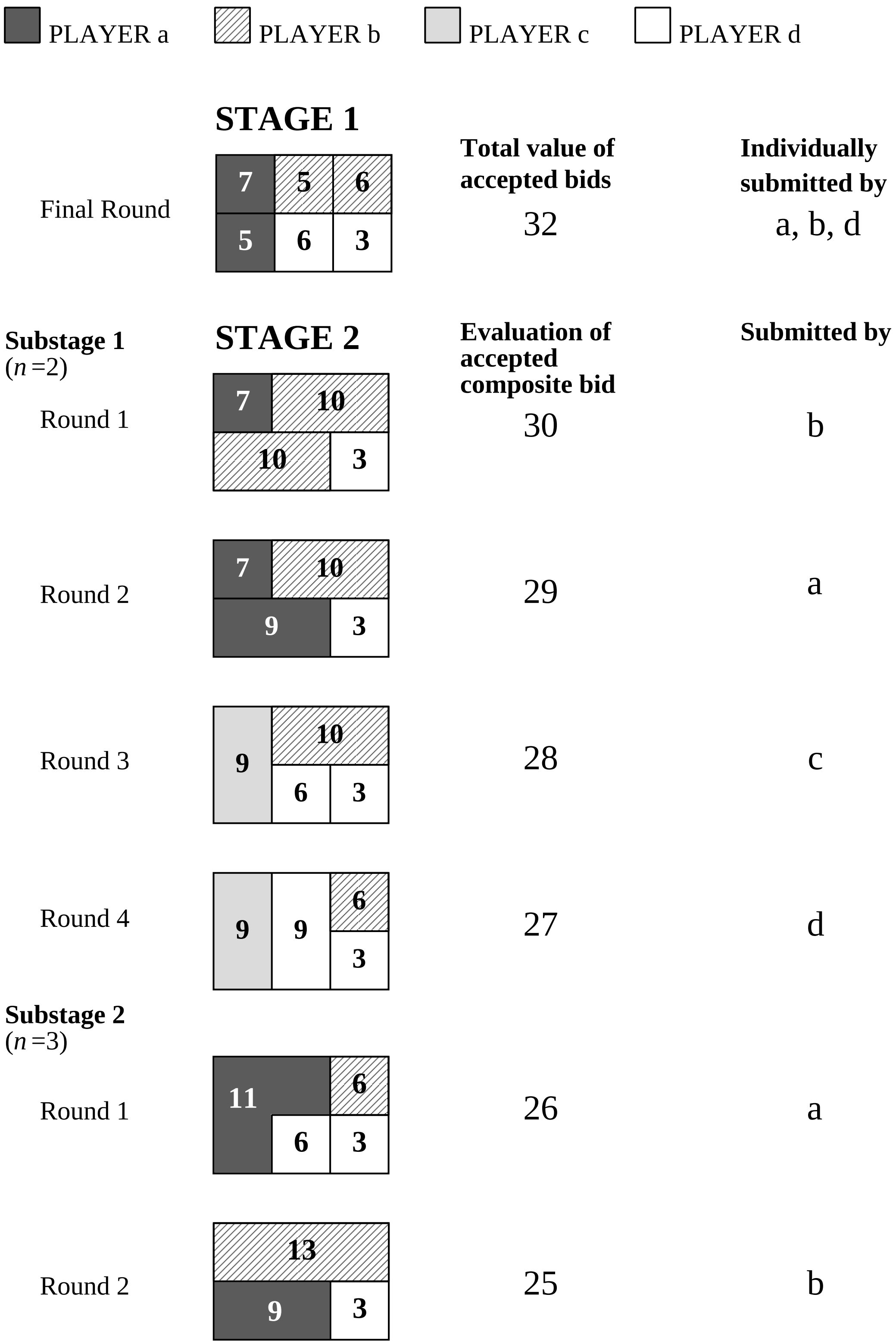
A player's winning bid on property j will, in general, be part of a composite bid. Thus, the limitation on the fraction of customers for which a player will be subsidized prevents the player from cross-subsidizing property j from the other properties that comprise his bid. Of course, each player is free to compete for any or all the customers in property j, although the player will not receive subsidy for any customers beyond the fraction he has won in the auction.

It is essential that, before the start of Stage 2, the auctioneer specifies the rules that need to be satisfied by a valid composite bid in a manner that can be checked by players, as well as by the auctioneer. In particular, the auctioneer should not attempt to decide the number of multiple winners after the conclusion of the auction, since to do so would involve the auctioneer in a task of some considerable computational complexity.

---

[12] This is in contrast to the AUSM scheme.

Figure 1 Illustration of Combinatorial Bidding



2.3 Other Auction Rules

Bid Waivers. The auction can include bid waivers, especially if the time between rounds is short. In Stage 1, the number of waivers would be concurrent with the number of waivers issued in the PCS auctions, and in Stage 2 a number 1.5 times this amount.

Bid Withdrawals. No bid withdrawals are allowed in either stage. It may be asked why they were permitted in the PCS auction, since they complicate the auction. P. Milgrom (1996a), in his attachment to GTE's comments, explains: "In effect, a bid withdrawal substitutes partially and quite imperfectly for combinatorial bidding." Porter (1999) reports results of experiments on auctions with bid withdrawals with penalties. He found that efficiency and revenue increase, but individual losses are larger. He also found that the increased efficiency does not outweigh the higher prices paid; thus, the bidder surplus falls.

More General Auctions. *Overlapping Bids.* For simplicity of exposition, we have assumed that the properties $j \in J$ are disjoint, but other possibilities may be of interest.[13]

Boolean Bids. A bid $c(p_i)$ is contingent in the sense that the bidder is offering to serve all the properties in the block p_i or none. More general forms of contingency could be allowed through Boolean bids. A Boolean bid is a string of blocks together with Boolean 'and', 'or' and bracket (left bracket or right bracket) connectors, together with a value or values. The format of the Boolean bid would need to be specified in such a manner that, given a set of properties, it would be an elementary computational task to evaluate the value of the bid for that set of properties (with the value being infinity if the set of properties is not consistent with the Boolean bid).

3. The Propositions

PAUSE clearly allows for all combinatorial bids and allows for multiple winners. Desiderata item (a) is addressed by requiring bidders in Stage 2 to submit composite bids. The auctioneer's procedure described in Section 2.2 for evaluating the combinatorial bids clearly is computationally tractable, assuming that the associated information storage problem is manageable. This is easily settled by Proposition 1, which addresses desiderata item (b). However, Proposition 1 first addresses desiderata item (c) by showing that the completion time of the auction is bounded. Propositions 2 and 3 respond to desiderata items (d) and (e), respectively.

Proposition 1. Bounds on Number of Rounds. Since in each round of Stage 2 the value of the accepted composite bid must decrease by at least ϵ over the previously accepted composite bid, the number of rounds in total is bounded above by $C_0(P_0)/\epsilon$, where $C_0(P_0)$ is the value of the opening composite bid.

Let B be the number of bidders. Since each bidder is allowed to make at most one composite bid per round, the maximum number of bids that needs to be registered by the auctioneer is bounded above by $\frac{C_0(P_0)}{\epsilon}B|J|$. QED.

---

[13] For example, in the auction of frequency bands for PCS, there may be two or more incompatible band plans, with a simple definition of disjoint properties within each band plan. In this case, Stage 1 could be used to set an initial price for each property in each band plan, with a composite bid in Stage 2 relating to a player's preferred band plan.

Remarks. Although for the procedure we present here, the auctioneer's problem of determining the winning bid is computationally tractable, for a bidder it may be an NP-complete problem to determine whether he can make a composite bid that beats the currently accepted composite bid. However, there is very little computational burden for small players interested in only a small number of properties. If no synergies are claimed, then the auction reduces to an auction of the type utilized for the PCS licenses. As discussed in the Introduction, the results of Rothkopf, Pekeč and Harstad (1998) show that, if the form of composite bids is restricted in one or other of several possible ways, then the problem can be made computationally tractable. However, in cases where the bidders are unlikely to agree upon the form of the appropriate restriction on composite bids, we view the elicitation of the form and size of potential synergies as a major purpose of the auction proposed here.

Work on computationally difficult problems shows that in several situations where finding the exact optimum is hard, finding a good approximation to the optimum with high probability may be relatively easy (Jerrum and Sinclair 1996). It is our belief that the traditional problems of elicitation and gaming are more serious difficulties than the possible computational burden on those bidders claiming complex synergies.

Proposition 2. Prevention against Jump Bidding and Mitigation of the Threshold Problem. The rule that the improvement margin is bounded above by 2ϵ reduces the possibility of price-jump bidding. The progression of allowable block size n reduces the possibility of block-jump bidding and mitigates the threshold problem. QED.

Remarks. The upper bound on the improvement margin also tends to reduce the bidders' computation requirements, by limiting the range of possibilities that need to be considered. The existence of the block size also aids the auctioneer in controlling the speed at which the auction progresses.

Proposition 3. Minimization of Price-Level Signaling. Price-level signaling is minimized by having the number of multiple winners determined prior to Stage 2; thus, most of the bidder surplus for the individual properties has already been extracted. Any additional surplus is most likely to be due to synergies, for which price-level signaling is very difficult. In addition, price-level signaling is very difficult to take advantage of, even if successful, due to the very nature of combinatorial bids, viz., that they involve more than one property, and they overlap in ways that usually cannot be easily disaggregated. QED.

Remarks. Since a fixed number of multiple winners can be accepted on a given individual property, and the contracts for each will carry the same obligations, then rational behavior by the bidders will generally lead to them achieving the same price (within ϵ) on successful bids on blocks comprising just that property. This is simply the law of one price, i.e., a bidder is unlikely to pay more for something identical available at a lower price. Note that a price for a property cannot be determined from a composite bid if, within that composite bid, the property is part of a larger block. Similarly, if the contracts carry different obligations, then rational behavior by the bidders will lead to a variety of achieved prices reflecting the bidders' views about the value to the bidders of the various obligations.

4. Implementation

Before the start of a PCS auction, each bidder is required to post a bond based on the number of licenses he hopes to win. A similar bonding procedure may be appropriate for a universal service auction. Opening bids for each property could be set as the lower of the historical cost and the forward-looking cost for that property. (See Leighton (1996) for details.) If the lower of these two costs is the historical cost, then it would be announced that historically service has been provided on this property at a certain subsidy level and it is expected that service will be provided at no higher than that level in the future. If, on the other hand, the lower of these two costs is the forward-looking cost, then forward-looking cost would serve as a starting point for the analysis to determine the minimum subsidy to provide service in a given market.

Note that no new bidder cost studies are required for participation in this auction. To participate, each firm needs to know only the value of its synergies, something that such a firm most likely already calculates and is part of the firm's information. The costs of auction administration will be minimal, as shown by Proposition 1, which provides bounds on the number of rounds and on the number of bids that need to be registered.

5. Testing and Validation

Like the PCS and AUSM auction structures, PAUSE is probably too complex to admit much theoretical analysis. However, to prove its value as a combinatorial bidding design, it could be tested and validated in an experimental setting. We suggest a two-phase testing procedure. In the first phase, the PAUSE auction can be refined by adjusting the parameters through simulation; general guidelines are provided. In the second phase, PAUSE can be tested against other auction approaches.

Testing—Phase 1: Refinement of the PAUSE auction. We suggest that simulation be used to adjust the following sets of parameters.

Eligibility Requirements: A_i ($i = 1, 2, 3$). (In the early PCS auctions these bounds were $(A_1, A_2, A_3) = (33, 67, 100)$. However, the first two bounds were found to be insufficiently stringent and were increased in later auctions.)

Bid Increments and Block Size: ϵ, and the progression of n. (For example, the auctioneer might move n from the starting value of 2, to 3, 4, 5, ...; however, the auctioneer might instead move n to 4, 8, 16,.... In either case the value of the bid increment would decrease, and the activity rule percentage increase, as n increases.)

Transition between Substages: T_i for all i, for each stage. (The FCC has found that, on occasion, it has been desirable to make *ad hoc* modifications to the T_i values.)

Outcome Rule: M_1 and M_2. (Milgrom (1996a) points out that the cut-off values he presents of 15 and 25 percent are illustrative only and not based on any detailed analysis.)

Testing—Phase 2: Comparison with other auction processes. Among the possible validation procedures, PAUSE could be tested by adapting the procedure of Rassenti, Smith and Bulfin (1982), called "RSB," and or that of Ledyard, Porter and Rangel (1997). Specifically, Rassenti, Smith and Bulfin (1982) made use of a $2 \times 2 \times 2$ experimental design, with the three factors being: (1) an alternative method (control) vs. RSB (treatment), (2) experience level of subject (experienced vs. inexperienced), and (3) combinatorial complexity of resource utilization ("easy" vs. "difficult"). Ledyard, Porter and Rangel (1997) examined the robustness of several auction mechanisms to allocate multiple objects. They compared three auction processes: (1) the simultaneous discrete process used in the PCS auction, (2) a sequential auction process, and (3) a continuous-time AUSM mechanism. Complete details can be found in their paper.

In this paper, we have focused on the computational challenges involved in combinatorial auctions, an essential step before testing. With the results of tests, it will be possible to explore the allocative efficiency of auction designs.

6. Conclusion

In March 1997, a preliminary version of this paper was submitted *ex parte* to the FCC by the Citizens for a Sound Economy Foundation (Kelly and Steinberg 1997). In March 1998, the FCC released a "Request for Proposal" (RFP) for a combinatorial bidding system (FCC 1998). It seems likely that this is the beginning of a period of substantial testing and validation of combinatorial auction mechanisms.

The proposal of this paper, while designed specifically for the United States, would apply with little or no modification to universal service obligation in other parts of the world. For example, in a recent paper Nett (1998) discusses the idea of using combinatorial auctions to allocate universal service obligation within the European Union. He presents a list of relevant criteria for such an auction; essentially all his criteria are addressed by the procedure described here.[14]

Appendix: Alternative Structure for Stage 1

The method by which the numbers of multiple winners are determined that is described in Section 2 is consistent with Milgrom (1996b), who showed that in a setting with a single property or where all the properties are independent (i.e., no synergies), an optimal auction design entails endogenous market structure. Here we describe a modification to PAUSE where the first stage is conducted as a sealed-bid auction.

The Bidders. Each bidder submits a *panel* of bids, made known only to the auctioneer, for each individual property on which he has an interest. The panel of bids on a given property from player i $(i = 1, 2, ..., L)$ is $(c(1, i), c(2, i), ..., c(M, i))$. Here $c(m, i)$ is the subsidy player i requires to be one of m multiple winners on the property and M is the maximum number of winners the auction authority will allow on the property. (We would expect the sequence $\{m \cdot c(m, i)\}$ to be increasing in m.)

The Auctioneer. The auctioneer generally prefers winners of greater multiplicity and will discount the bids accordingly. Let $f(m)$ be the auctioneer's *m-competitor discount factor*. (Here, $f(1) \equiv 0$, and we would expect the function $f(m)$ to be increasing in m.)

Multiple Winners. After all the sealed bids have been submitted, the auctioneer computes for each property,[15] $C(m) = m \cdot c'(m)$, where $c'(m) = m$-th smallest term in the set $S(m) = \{c(m, 1), c(m, 2), ..., c(m, L)\}$, for $m = 1, 2, ..., M$. The auctioneer next computes, for each property, $m^* = \arg\min\{(1 - f(m)) \cdot C(m)\}$. The auctioneer announces m^* as the number of multiple winners for that property, and the bids from

---

[14] We are grateful to Richard Gibbens, Ron Harstad, Evan Kwerel, Wayne Leighton, and the editors and reviewers for valuable comments on earlier drafts.

[15] See Vickery (1962). The auction we describe is more general than that considered by Vickery, viz., it is an example of a "menu" auction. (See Bernheim and Whinston 1986.)

the set $S(m)$ achieving at least $C(m^*)/m^*$ as the Stage 1 winning bids, each at the subsidy level $C(m^*)/m^*$. The number of multiple winners on each property j at the end of Stage 1 is denoted by $m(j)$. Before the start of Stage 2, property j is replaced by $m(j)$ properties $j_1, j_2, j_3, \ldots, j_{m(j)}$, each allocated a nominal number of subscribers equal to $\mathrm{sub}(j)/m(j)$, where $sub(j)$ denotes the numbers of subscribers in property j.

Asymmetries and Closed-Bid Auctions. An important recent contribution to the debate about the relative merits of first- and second-price auctions has been made by Klemperer (1998), who has argued that small asymmetries between bidders may substantially lower the price achieved in auctions for "almost common-value" objects such as licenses in "airwave auctions." We have been influenced by Klemperer's arguments to define the modification of Stage 1 in terms of "last accepted" as opposed to "first rejected" bids.

References

Banks, J.S., J.O. Ledyard, D. Porter. 1989. Allocating uncertain and unresponsive resources: an experimental approach. *Rand Journal of Economics* **20** 1–25.

Bernheim, B.D., M. Whinston 1986. Menu auctions, resource allocation, and economic influence. *Quarterly Journal of Economics* **101** 1-31.

Bykowsky, M.M., R.J. Cull. 1993. Issues in implementing a personal communications services auction. Attached to Comments of the National Telecommunications and Information Administration in PP Docket No. 93–253.

Cramton, P. 1997. The FCC Spectrum Auctions: An Early Assessment. *Journal of Economics & Management Strategy* **6** 431-495.

The Economist. 1997. Learning to play the game. 17 May, 120.

Edmonds, J. 1965. Maximum matching and a polyhedron with 0,1 vertices. *Journal of Research of the National Bureau of Standards* **69B** 125–130.

Federal Communications Commission. 1993. In the matter of implementation of Section 309(j) of Communication Act, Competitive Bidding: Notice of Proposed Rule Making, PP Docket No. FCC 93–253, Washington, D.C.

_____. 1997. Third Report and Order and Second Further Notice of Proposed Rulemaking, Washington, D.C.

_____. 1998. Combinatorial Bidding System. Solicitation S–9854013, Washington, D.C.

Grether, D.M., R.M. Isaac, C.R. Plott. 1989. *The Allocation of Scarce Resources: Experimental Economics and the Problem of Allocating Airport Slots*, Westview Press, Boulder, Colorado.

Jerrum, M., A. Sinclair. 1996. The Markov Chain Monte Carlo method: an approach to approximate counting and integration, in *Approximation Algorithms for NP-Hard Problems* (ed. Dorit S. Hochbaum), PWS Publishing Company, Boston, Massachusetts.

Karp, R.M. 1972. Reducibility among combinatorial problems, in *Complexity of Computer Computations* (ed. R.E. Miller and J.W. Thatcher), Plenum Press, New York, pp. 85–103.

Kelly, F., R. Steinberg. 1997. A Combinatorial Auction with Multiple Winners for COLR. University of Cambridge, 17 March. [Submitted *ex parte* 18 March 1997 by Citizens for a Sound Economy Foundation to US Federal Communications Commission re CC Docket No. 96–45, Federal-State Board on Universal Service.]

Klemperer, P. 1998. Auctions with almost common values: The ‘Wallet Game’ and its applications. *European Economic Review* **42** 757-769.

Kwerel, E., A.D. Felker. 1985. Using Auctions to Select FCC Licensees. Office of Plans and Policy Working Paper No. 16, Federal Communications Commission, May.

Ledyard, J.O., D. Porter, A. Rangel. 1997. Experiments testing multiobject allocation mechanisms. *Journal of Economics & Management Strategy* **6** 639–675.

Leighton, W. 1996. What to do about Universal Service Subsidies: Support for High-Cost Areas, Issue Analysis 39 (September 30). Citizens for a Sound Economy Foundation, Washington, D.C.

McAfee, R.P. 1993. Auction Design for Personal Communications Services: Reply Comments. PacTel Exhibit in PP Docket No. 93–253.

_____, J. McMillan. 1996. Analyzing the airwaves auction. *Journal of Economic Perspectives* **10** 159–175.

McCabe, K.A., S.J. Rassenti, V.L. Smith. 1991. Smart computer assisted markets. *Science* **254** 534–538.

Milgrom, P.R. 1996a. Statement attached to GTE's Comments in response to questions. Federal Communications Commission, Notice of Proposed Rulemaking, Federal-State Joint Board on Universal Service, CC Docket No. 96–45.

_____ 1996b. Procuring Universal Service: Putting Auction Theory to Work. Lecture at the Royal Swedish Academy of Sciences, Stockholm, Sweden, December 9.

Nett, L. 1998. Auctions: An alternative approach to allocate universal service obligations. *Telecommunications Policy* **22** 661-669.

Omnibus Budget Reconciliation Act. 1993. Pub. L. No. 103–66, 107 Stat. 312.

Porter, D.P. 1999. The Effect of Bid Withdrawal in a Multi-Object Auction, *Review of Economic Design* **4** 73–97.

Rassenti, S.J., V.L. Smith, R.L. Bulfin. 1982. A combinatorial auction mechanism for airport time slot allocation. *Bell Journal of Economics* **13** 402–417.

Rothkopf, M.H., A. Pekeč, R.M. Harstad. 1998. Computationally Manageable Combinational Auctions. *Management Science* **44** 1131-1147.

Telecommunications Act of 1996. Pub. LA. No. 104–104, 110 Stat. 56.

Vickery, W. 1962. Auctions and Bidding Games, in: *Recent Advances in Game Theory* Princeton University Press, Princeton, New Jersey.