

Cambridge Compromise

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*On the apportionment of the seats in
the European Parliament:
A report by mathematicians*

Provisional recommendations

AFCO 7 February 2011

Mathematics

Prof. Geoffrey Grimmett (University of Cambridge)

Prof. Friedrich Pukelsheim (University of Augsburg)

Prof. Jean-François Laslier (École Polytechnique, Paris)

Prof. Victoriano Ramírez González (University of Granada)

Prof. Wojciech Słomczyński (Jagiellonian University, Cracow)

Prof. Martin Zachariasen (University of Copenhagen)

Prof. Karol Życzkowski (Jagiellonian University, Cracow)

Public Policy

Prof. Richard Rose (University of Aberdeen)

Students in attendance

Mr Thomas Kellermann (College of Europe, Natolin)

Ms Kai-Friederike Oelbermann (University of Augsburg)

AFCO in attendance

Mr Andrew Duff MEP (Rapporteur)

Mr Rafał Trzaskowski MEP (Vice-President of AFCO)

Mr Wolfgang Leonhardt (Administrator of AFCO)

Mr Guy Deregnaucourt (Administrator of AFCO)

Mr Kevin Wilkins (Assistant to Mr Duff)

from the Rapporteur's "Note":

"To discuss and, if possible, to propose to the Committee on Constitutional Affairs a **mathematical formula** for the redistribution of the 751 seats in the European Parliament. The formula should be as **transparent** as possible and capable of being **sustained** from one Parliamentary mandate to the next.

"The purpose of the reform is to eliminate the political bartering which has characterised the distribution of seats so far by enabling a smooth reallocation of seats once every five years which takes into account **migration**, **demographic shifts** and the **accession** of new member states."

- ▶ Parliament-size shall not exceed 751,
- ▶ minimum threshold of 6 seats per Member State,
- ▶ no Member State shall receive more than 96 seats,
- ▶ no smaller State shall receive more seats than a larger State,
- ▶ the allocation shall respect “**degressive proportionality**”.

Lamassoure–Severin interpretation

... each Member from a more populous State represents more citizens than each Member from a less populous State ...

Cambridge proposal

The ratio between the population and the number of seats **before rounding to whole numbers** of each State must vary ... in such a way that each Member from a more populous State represents more citizens than each Member from a less populous State ...

Why is the LS condition too demanding?

- ▶ there exist instances of the apportionment problem (with fixed house-size) having **no solution**,
- ▶ ● blocks of States with similar populations can be forced to have equal numbers of seats, and
 - the accession of a new state can impose equality over a greater range of populations.

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 - the accession of a new state can impose equality over a greater range of populations.

A proposal is sought that is:

- ▶ **durable** — *enlargement, migration, demographic shifts,*
- ▶ **transparent** — *capable of simple and reasonable explanation,*
- ▶ **impartial** to politics.

Observations:

- ▶ EU has currently 27 Member States,
- ▶ smallest population is 412 970, largest 81 802 257,
- ▶ future accessions may include a number of small States and perhaps larger States,
- ▶ there will be migration and demographic changes,
- ▶ national statistics (as available via Eurostat) will be used as input to the formula.

Base+prop allocation

1. assign to each Member State a fixed number of seats, called the *base* and denoted b ,
2. for a given *divisor* d , assign to a State with population p a further quotient p/d of *notional* seats,
3. perform a *rounding* of the quotients, replacing the quotient p/d by a whole number $[p/d]$,
4. if the seat total $b + [p/d]$ exceeds the maximum allocation, replace it by this maximum,
5. adjust the divisor d in such a way that the total number of seats equals the given Parliament-size.

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- (i) set the base $b = 5$,
- (ii) round *upwards*.

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Bases and Rounding methods

- ▶ a **smaller** base?
- ▶ a **larger** base?

- ▶ round **downwards**: $5.1 \downarrow 5$, $5.9 \downarrow 5$, $6 \rightarrow 6$,
- ▶ round to **nearest integer**: $5.1 \downarrow 5$, $5.9 \uparrow 6$, $6 \rightarrow 6$
- ▶ round **upwards**: $5.1 \uparrow 6$, $5.9 \uparrow 6$, $6 \rightarrow 6$.

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Bases and Rounding methods

- ▶ a **smaller** base *tends to favour larger States,*
- ▶ a **larger** base *tends to favour smaller States.*

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tends to favour larger States,
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generally regarded as fairly neutral to State-size,
- ▶ round **upwards**: $5.1 \uparrow 6$, $5.9 \uparrow 6$, $6 \rightarrow 6$
tends to favour smaller States.

m := minimal number of seats for any State

M := upper bound of seats for any State

H := Parliament size.

Member State i , with population p_i , receives:

$$s_i := \min \left\{ b + \lceil p_i/d \rceil, M \right\}$$

where d is chosen so that $\sum_i s_i = H$.

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- ▶ $m = 6$, $M = 96$, $H = 751$,
- ▶ $b = 5$,
- ▶ $\lceil x \rceil$ denotes x rounded upwards.

m := minimal number of seats for any State

M := upper bound of seats for any State

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Member State i , with population p_i , receives:

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Row	Member State <i>OJ 22.12.2010 L 338/47</i>	Population	Base + Quotient → Seats <i>Population</i> 819000	<i>B+Q ratio</i> <i>Population</i> Base+Quot	<i>S ratio</i> <i>Population</i> Seats	Now
1	Germany	81 802 257	5 + 99.9 ↓ 96	779 955.6	852 106.8	99
2	France	64 714 074	5 + 79.02 ↑ 85	770 259.3	761 342.0	74
3	United Kingdom	62 008 048	5 + 75.7 ↑ 81	768 264.0	765 531.5	73
4	Italy	60 340 328	5 + 73.7 ↑ 79	766 950.8	763 801.6	73
5	Spain	45 989 016	5 + 56.2 ↑ 62	752 036.4	741 758.3	54
6	Poland	38 167 329	5 + 46.6 ↑ 52	739 643.2	733 987.1	51
7	Romania	21 462 186	5 + 26.2 ↑ 32	687 772.5	670 693.3	33
8	Netherlands	16 574 989	5 + 20.2 ↑ 26	656 745.2	637 499.6	26
9	Greece	11 305 118	5 + 13.8 ↑ 19	601 222.1	595 006.2	22
10	Belgium	10 839 905	5 + 13.2 ↑ 19	594 438.5	570 521.3	22
11	Portugal	10 637 713	5 + 12.99 ↑ 18	591 356.6	590 984.1	22
12	Czech Republic	10 506 813	5 + 12.8 ↑ 18	589 315.9	583 711.8	22
13	Hungary	10 014 324	5 + 12.2 ↑ 18	581 298.7	556 351.3	22
14	Sweden	9 340 682	5 + 11.4 ↑ 17	569 380.7	549 451.9	20
15	Austria	8 375 290	5 + 10.2 ↑ 16	550 056.4	523 455.6	19
16	Bulgaria	7 563 710	5 + 9.2 ↑ 15	531 334.8	504 247.3	18
17	Denmark	5 534 738	5 + 6.8 ↑ 12	470 724.2	461 228.2	13
18	Slovakia	5 424 925	5 + 6.6 ↑ 12	466 706.8	452 077.1	13
19	Finland	5 351 427	5 + 6.5 ↑ 12	463 965.8	445 952.2	13
20	Ireland	4 467 854	5 + 5.5 ↑ 11	427 330.9	406 168.5	12
21	Lithuania	3 329 039	5 + 4.1 ↑ 10	367 250.6	332 903.9	12
22	Latvia	2 248 374	5 + 2.7 ↑ 8	290 290.0	281 046.8	9
23	Slovenia	2 046 976	5 + 2.5 ↑ 8	272 953.4	255 872.0	8
24	Estonia	1 340 127	5 + 1.6 ↑ 7	201 939.0	191 446.7	6
25	Cyprus	803 147	5 + 0.98 ↑ 6	134 291.1	133 857.8	6
26	Luxembourg	502 066	5 + 0.6 ↑ 6	89 446.6	83 677.7	6
27	Malta	412 970	5 + 0.5 ↑ 6	75 027.7	68 828.3	6
	<i>Total</i>	<i>501 103 425</i>	<i>135</i>	<i>751</i>		<i>754</i>

Each 819 000 Union citizens or part thereof account for one of the 616 remaining seats (except Germany).
B+Q ratios are strictly decreasing, as are S ratios except for the four regresses in shaded cells.

Row	Member State	Population	Base + Quotient	→ Seats	B+Q ratio	S ratio	Now
	Eurostat, as of 1.1.2011		Population		Population	Population	
			835 000		Base+Quot	Seats	
1	Germany	81 802 257	5 + 97.97	↓ 96	794 452.9	852 106.8	99
2	France	64 714 074	5 + 77.5	↑ 83	784 395.1	779 687.6	74
3	United Kingdom	62 008 048	5 + 74.3	↑ 80	782 326.0	775 100.6	73
4	Italy	60 340 328	5 + 72.3	↑ 78	780 964.4	773 593.9	73
5	Spain	45 989 016	5 + 55.1	↑ 61	765 505.5	753 918.3	54
6	Poland	38 167 329	5 + 45.7	↑ 51	752 668.1	748 379.0	51
7	Romania	21 462 186	5 + 25.7	↑ 31	699 020.8	692 328.6	33
8	Netherlands	16 574 989	5 + 19.9	↑ 25	666 993.9	662 999.6	26
9	Greece	11 305 118	5 + 13.5	↑ 19	609 799.8	595 006.2	22
10	Belgium	10 839 905	5 + 12.98	↑ 18	602 822.4	602 216.9	22
11	Portugal	10 637 713	5 + 12.7	↑ 18	599 653.2	590 984.1	22
12	Czech Republic	10 506 813	5 + 12.6	↑ 18	597 554.9	583 711.8	22
13	Hungary	10 014 324	5 + 11.99	↑ 17	589 313.5	589 077.9	22
14	Sweden	9 340 682	5 + 11.2	↑ 17	577 068.1	549 451.9	20
15	Austria	8 375 290	5 + 10.03	↑ 16	557 227.5	523 455.6	19
16	Bulgaria	7 563 710	5 + 9.1	↑ 15	538 023.2	504 247.3	18
17	Denmark	5 534 738	5 + 6.6	↑ 12	475 966.1	461 228.2	13
18	Slovakia	5 424 925	5 + 6.5	↑ 12	471 859.1	452 077.1	13
19	Finland	5 351 427	5 + 6.4	↑ 12	469 057.4	445 952.2	13
20	Ireland	4 467 854	5 + 5.4	↑ 11	431 646.5	406 168.5	12
21	Croatia	4 425 747	5 + 5.3	↑ 11	429 671.8	402 340.6	–
22	Lithuania	3 329 039	5 + 3.99	↑ 9	370 433.5	369 893.2	12
23	Latvia	2 248 374	5 + 2.7	↑ 8	292 275.1	281 046.8	9
24	Slovenia	2 046 976	5 + 2.5	↑ 8	274 707.7	255 872.0	8
25	Estonia	1 340 127	5 + 1.6	↑ 7	202 897.6	191 446.7	6
26	Cyprus	803 147	5 + 0.96	↑ 6	134 714.3	133 857.8	6
27	Luxembourg	502 066	5 + 0.6	↑ 6	89 634.2	83 677.7	6
28	Malta	412 970	5 + 0.5	↑ 6	75 159.6	68 828.3	6
	Total	505 529 172	140	751			754

Each 835 000 Union citizens or part thereof account for one of the 611 remaining seats (except Germany).
B+Q ratios are strictly decreasing, as are S ratios except for the two regresses in shaded cells.

Row	Member State	Population	Base + Quotient → Seats	B+Q ratio	S ratio	Now
	Eurostat, as of 1.1.2011			$\frac{\text{Population}}{\text{Base + Quot}}$	$\frac{\text{Population}}{\text{Seats}}$	
1	Germany	81 802 257	5 + 96.9 ↓ 96	802 595.8	852 106.8	99
2	France	64 714 074	5 + 76.7 ↑ 82	792 332.1	789 196.0	74
3	United Kingdom	62 008 048	5 + 73.5 ↑ 79	790 221.0	784 912.0	73
4	Italy	60 340 328	5 + 71.5 ↑ 77	788 831.8	783 640.6	73
5	Spain	45 989 016	5 + 54.5 ↑ 60	773 062.9	766 483.6	54
6	Poland	38 167 329	5 + 45.2 ↑ 51	759 973.0	748 379.0	51
7	Romania	21 462 186	5 + 25.4 ↑ 31	705 317.1	692 328.6	33
8	Netherlands	16 574 989	5 + 19.6 ↑ 25	672 724.1	662 999.6	26
9	Greece	11 305 118	5 + 13.4 ↑ 19	614 586.0	595 006.2	22
10	Belgium	10 839 905	5 + 12.8 ↑ 18	607 499.2	602 216.9	22
11	Portugal	10 637 713	5 + 12.6 ↑ 18	604 280.7	590 984.1	22
12	Czech Republic	10 506 813	5 + 12.4 ↑ 18	602 150.0	583 711.8	22
13	Hungary	10 014 324	5 + 11.9 ↑ 17	593 782.3	589 077.9	22
14	Sweden	9 340 682	5 + 11.1 ↑ 17	581 352.4	549 451.9	20
15	Austria	8 375 290	5 + 9.9 ↑ 15	561 221.3	558 352.7	19
16	Bulgaria	7 563 710	5 + 8.96 ↑ 14	541 745.4	540 265.0	18
17	Denmark	5 534 738	5 + 6.6 ↑ 12	478 876.9	461 228.2	13
18	Slovakia	5 424 925	5 + 6.4 ↑ 12	474 719.8	452 077.1	13
19	Finland	5 351 427	5 + 6.3 ↑ 12	471 884.1	445 952.2	13
20	Ireland	4 467 854	5 + 5.3 ↑ 11	434 039.2	406 168.5	12
21	Croatia	4 425 747	5 + 5.2 ↑ 11	432 042.5	402 340.6	–
22	Lithuania	3 329 039	5 + 3.9 ↑ 9	372 194.3	369 893.2	12
23	Latvia	2 248 374	5 + 2.7 ↑ 8	293 370.1	281 046.8	9
24	Slovenia	2 046 976	5 + 2.4 ↑ 8	275 674.9	255 872.0	8
25	Estonia	1 340 127	5 + 1.6 ↑ 7	203 424.7	191 446.7	6
26	Cyprus	803 147	5 + 0.95 ↑ 6	134 946.5	133 857.8	6
27	Luxembourg	502 066	5 + 0.6 ↑ 6	89 736.9	83 677.7	6
28	Malta	412 970	5 + 0.5 ↑ 6	75 231.8	68 828.3	6
29	Iceland	317 630	5 + 0.4 ↑ 6	59 079.2	52 938.3	–
	Total	505 846 802	145	751		754

Each 844 000 Union citizens or part thereof account for one of the 606 remaining seats (except Germany).
B+Q ratios are strictly decreasing, as are S ratios except for the three regresses in shaded cells.

- ▶ If **Croatia** were to accede during the next legislative period, the European Parliament could be temporarily enlarged by $5 + 6 = 11$ seats (since $4\,425\,747/819\,000 = 5.4 \uparrow 6$).
- ▶ If **Iceland** were to accede during the following legislative period, the European Parliament could be temporarily enlarged by $5 + 1 = 6$ seats (since $317\,630/835\,000 = 0.4 \uparrow 1$).

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Two classes of method

- ▶ non-linear functions: parabolic, power, ...
- ▶ linear (affine) functions: base+prop, spline, ...

Base+prop allocations

- ▶ base = 6, rounding to nearest integer,
- ▶ base = 5, rounding upwards.

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Base+prop allocations

- ▶ base = 6, rounding to nearest integer,
- ▶ base = 5, rounding upwards.

Why 5+ rounding upwards?

	<i>Member State</i>	<i>Population</i>	<i>5 + upwards</i>	<i>6+ nearest</i>	<i>Now</i>
24	Estonia	1 340 127	7	8	6
25	Cyprus	803 147	6	7	6
26	Luxembourg	502 066	6	7	6
27	Malta	412 970	6	6	6

Why 5+ rounding upwards?

	<i>Member State</i>	<i>Population</i>	<i>5 + upwards</i>	<i>6+ nearest</i>	<i>Now</i>
24	Estonia	1 340 127	7	8	6
25	Cyprus	803 147	6	7	6
26	Luxembourg	502 066	6	7	6
27	Malta	412 970	6	6	6

Revised allocations if Malta were to grow by **8 000**:

	<i>Member State</i>	<i>Population</i>	<i>5 + upwards</i>	<i>6+ nearest</i>	<i>Now</i>
24	Estonia	1 340 127	7	8	6
25	Cyprus	803 147	6	7	6
26	Luxembourg	502 066	6	7	6
27	Malta	420 970	6	7	6

Degressive proportionality revisited

“10. Within this context, how to go forward? The ideal alternative would be to agree on an **undisputed mathematical formula** of “degressive proportionality” that would **ensure a solution** not only for the present revision but for future enlargements or modifications due to demographic changes.”

On the composition of the European Parliament (2007/2169(INI))

Degressive proportionality revisited

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On the composition of the European Parliament (2007/2169(INI))

A five-State example with no solution:

<i>Member State</i>	<i>Population (2010)</i>	<i>Apportionment</i>
Greece	11 305 118	21
Belgium	10 839 905	21
Portugal	10 637 713	21
Czech Republic	10 506 813	21
Hungary	10 014 324	21
<i>Totals</i>	<i>53 303 873</i>	<i>105</i>

What if the Parliament-size is fixed at 106?

Degressive proportionality revisited

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<i>Totals</i>	<i>53 303 873</i>	<i>105</i>

What if the Parliament-size is fixed at 106?