NEW FRONTIERS IN RANDOM GEOMETRY (RaG)
EP/I03372X/1
REPORT 1/7/15 – 30/6/16

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1. Management Process

The Management Committee (MC) comprises the three investigators and the four members of the external Advisory Board (AB), namely Yuval Peres, Stanislav Smirnov, Balint Tóth and Wendelin Werner. The local managers have met weekly during term, and more formally about every two months. The advice of the AB has been sought on a variety of matters including the hiring process. Two members of the AB (Peres, Werner) have spent periods in Cambridge during the period of this report. A meeting of the Advisory Board is due later in July 2016.

2. Personnel

One postdoctoral research fellow was appointed following the advertisement of December 2015.

- Marcin Lis[1], PhD (VU Amsterdam), from 1 October 2016 to 31 August 2017.

One postdoc has left the team, and another will depart later in the summer of 2016:

- Laure Dumaz[2], employed from 1 September 2013 to 31 August 2015,
- Benoît Laslier[3], employed from 1 September 2014 to 31 August 2016.

3. Scientific meetings

In addition to the weekly seminar, members of the group organised a successful week of concentration on aspects of dimers, including a tutorial series of lectures by Julien Dubédat, and culminating on 23 May 2016 in a *Dimer Day*[4].

Date: July 16, 2016.
http://www.statslab.cam.ac.uk/~grg/rag.html.

[1] http://http://www.chalmers.se/en/Staff/Pages/marcinl.aspx

[2] https://www.dpmms.cam.ac.uk/people/ld437/

[3] http://www.statslab.cam.ac.uk/~bl395/

[4] http://www.statslab.cam.ac.uk/~beresty/DimerDay/index.html

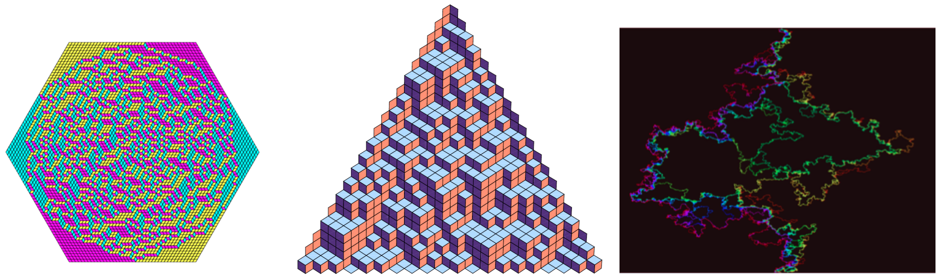

FIGURE 4.1. Left and center: dimer model (or equivalently lozenge tilings) with two different boundary conditions. Right: Flow lines of the GFF with $\kappa = 2$, as arising in the proof by Berestycki, Laslier and Ray.

4. Research Programme (selected highlights)

This year has been partly one of consolidation following our enormously successful six-month programme entitled *Random Geometry*, held at the Isaac Newton Institute from January to June 2015. Details of the programme's activities may be found on the INI website at http://www.newton.ac.uk/event/rgm. Some highlights of the last year's activities follow.

4.1. **Universality of height fluctuations in the dimer model.** The dimer model on a finite bipartite planar graph is a uniformly chosen set of edges which cover every vertex exactly once. It is a classical model of statistical mechanics, going back to work of Kasteleyn, Fisher and Temperley in the 1960s who computed its partition function.

The main question concerns the large-scale behaviour of the associated height function. For certain special domains of the square lattice, Kenyon (2000) had shown that the limit is a Gaussian Free Field. Berestycki, Laslier and Ray have developed a new method to study the fluctuations of the height function, which allows us to establish universality of the GFF in a variety of contexts. A key novelty in this approach is that the exact solvability of the model plays only a minor role. Instead, the proof relies on a connection to imaginary geometry, where Schramm–Loewner Evolution curves are interpreted as flow lines of an underlying Gaussian free field. Since it is the properties of the objects in the continuum which matter (as opposed to exact identities at the discrete level), the proof is robust and opens up many new possibilities for studying the dimer model beyond the square lattice.

4.2. **Tilted interface in low temperature 3D Ising model.** At low temperature, the Ising model has two symmetrical distinct translation invariant measures called the two pure phases. In a finite volume, it is possible to force the coexistence of these two phases and in $\mathbb{Z}^3$ an essentially two dimensional interface appear. When this interface is along a lattice direction, the local behaviour of the interface and its fluctuations have been known for a long time but nothing is known when the interface is in a generic position. Coquille and Laslier are investigating the latter case via a coupling with the zero temperature case (lozenge tilings). Preliminary results seem to indicate an exponential decay of correlation and logarithmic fluctuations of the interfaces.

4.3. **Hydrodynamic limit for the Luby–Randall–Sinclair dynamics on lozenge tilings.** In many dynamical models of statistical physics, it is believed that at large scale a kind of time-dependent law of large numbers called a *hydrodynamic limit* should appear. For example, for the dimer model, and any reasonable dynamics, the height function should evolve in an almost deterministic manner at large scale and actually satisfy a PDE. However in most cases, proving this seems beyond hope and even just finding the correct limit equation might not be doable.

In the case of a dynamics on lozenge tilings introduced by Luby–Randall–Sinclair, Laslier and Toninelli found a summation-by-parts formula that leads to a heuristic derivation of the equation for the hydrodynamic limit. The equation is a fully explicit parabolic non-linear PDE, of which they have derived some basic properties. They are currently investigating whether they can derive the full hydrodynamic limit.

4.4. **Rohde–Schramm theorem, via the Gaussian Free Field.** The Rohde–Schramm theorem is the foundational result in the SLE theory which asserts that SLE exists as a random curve, almost surely (that is, the solution of the Loewner equation driven by Brownian motion is generated by a curve almost surely). The proof of this result is very peculiar: the SLE estimates by Rohde and Schramm work for all values κ except $\kappa = 8$, in which case the result is only known as a consequence of convergence of Loop-Erased Random Walk (LERW) to SLE_2 by Lawler, Schramm and Werner, together with Wilson's algorithm which relates the Uniform Spanning Tree (UST) to LERW. Since the UST is obviously generated by a curve, so must SLE_8 in the limit! It has been an open question since that paper to come up with a unified "continuous" proof which does not require any special "discrete" arguments for $\kappa = 8$.

Berestycki and Jackson are currently finalising a proof of the Rohde–Schramm which uses a coupling to an underlying Gaussian Free Field (the "reverse" coupling of Liouville quantum gravity) to establish properties of the SLE through GFF estimates, which are typically easier to establish. At the moment, the work yields an alternative proof of the Rohde–Schramm theorem exactly in the same case as the original proof by Rohde and Schramm (namely, all values of κ except $\kappa = 8$).

4.5. **Random GUE matrices and Gaussian multiplicative chaos.** Links between GUE random matrices and random planar maps (or Liouville quantum gravity) are well known in the mathematical physics community, at least since the work of Itzykhson giving asymptotics for certain planar maps in terms of certain random matrix integrals.

In parallel, it has been known for some time that the logarithm of the characteristic polynomial of GUE random matrices converges to a log-correlated field in 1 dimension, which can be thought of as the restriction to the real line of the Neumann GFF in the half-plane.

Ongoing work by Berestycki, Webb and Wong show that this result can be strengthened in the sense that one can exponentiate this convergence result. This relates the characteristic polynomial to the random measures constructed by the theory of Gaussian multiplicative chaos (i.e., the boundary Liouville quantum gravity measure). Mathematically, the proof relies on an analysis of a Riemann–Hilbert problem and the method of steepest descent of Deift and Zhou.

4.6. **Penalised random walk.** A conjecture of Bolthausen (1994) is that a random walk trajectory, penalised by $\exp(-|R_n|)$ where R_n is the range of the walk at time n, is localised on a Euclidean ball of radius $\rho_d n^{1/(d+2)}$ where ρ is a certain deterministic constant. Bolthausen proved this result for the case $d = 2$ while the case $d \geq 3$ remains open.

In work in progress by Berestycki and Cerf, we resolve the conjecture. A key step is to use recent discrete Faber–Krahn inequalities in dimension $d \geq 3$. One highly nontrivial technical issue is that such inequalities are only known in the full plane and not in the torus. This makes it impossible to project random walk on the torus and make use of the Donsker–Varadhan theory (which requires a compact state space). Hence a by-product of our analysis is an extension of this theory to all of $\mathbb{Z}^d$, which should have independent applications.

4.7. **Scaling limits of random maps with boundary: a panorama.** Ray, along with Baur and Miermont, is working on the scaling limits of uniform planar quadrangulations with a boundary. Under different regimes of boundary size, bulk size and scaling parameter, various different limits emerge. This includes the well-known Brownian map and disc. Several new objects like the Brownian half plane, Brownian half plane with boundary drift, and infinite volume Brownian disc also are defined as scaling limits. Understanding the relationships between these and how to glue them together is a key question to be addressed in future work.

4.8. **Hyperbolic and parabolic random maps.** Ray, together with Angel, Hutchcroft and Nachmias, is finishing a draft on the dichotomy theorem for unimodular random planar maps. They introduce a notion of "average curvature" to classify a unimodular map as parabolic or hyperbolic. They also show that this classification is equivalent to various other geometric, probabilistic, analytic and conformal properties.

4.9. **The half-plane UIPT is recurrent.** Ray and Angel have completed their work on proving that the half-plane UIPT is recurrent. This involves constructing a new map with some nice stationarity properties, and understanding how their circle packing behaves.

4.10. **Contraction percolation.** Ray, with Georgakopoulos and Carmesin, is working on understanding how random walk on a graph behaves when one contracts subcritical percolation clusters. In the critical case, Benjamini, Kozma and Gurel-Gurevich have proved in an unpublished work that the contracted graph is transient.

4.11. **Yang–Mills measure and master field on the sphere.** Dahlqvist and Norris are investigating the behaviour of random Euclidean Yang–Mills fields, indexed by loops on compact surfaces and taking values in compact groups of matrices of high dimension. Such a study was undertaken by Lévy for the disc, who highlighted a limit object called the planar master field. These works are motivated by conjectures of Singer, based on the physics literature, claiming that such an object exists for any surface. When the surface is a sphere, Dahlqvist and Norris prove that the statement holds true and that the model admits a phase transition as the area of the sphere reaches a threshold. Up to the threshold area, the limit is described in a simple way thanks to the classical semicircle law. A key tool is the Makeenko–Migdal equation allowing to prove the result if known for simple loops. Another feature of the study is to show a duality formula relating this random matrix model to a discrete Coulomb gas on $\mathbb{Z}$ that applies to the field restricted to simple loops. The phase transition is then triggered by the saturation of this model of particles.

4.12. **Brownian map and Liouville quantum gravity.** Sheffield and Miller have continued their work on the relationship between the Brownian map (i.e., the limit of rescaled random planar maps viewed as random metric spaces) and Liouville quantum gravity with parameter $\gamma = \sqrt{8/3}$. More precisely, they have established the existence of a random metric on the sphere (deriving from Liouville quantum gravity with this parameter), and an isometry between the sphere equipped with this random metric, and the Brownian map. In previous work, the random metric has been defined on a dense countable set of points using the QLE (quantum Loewner evolution) process. Hence the main contribution of the paper is to show that this metric is "continuous" and extends to all of the sphere, and to show that it agrees in law with the Brownian map.

4.13. **1-2 model.** Grimmett and Li have continued their study of the phase transition of the 1-2 model. Their original exact identification of the critical surface of the homogeneous 1-2 model on the hexagonal lattice may be extended to non-homogeneous periodic models. They have derived a new methodology which complements that of their earlier papers, and proceeds

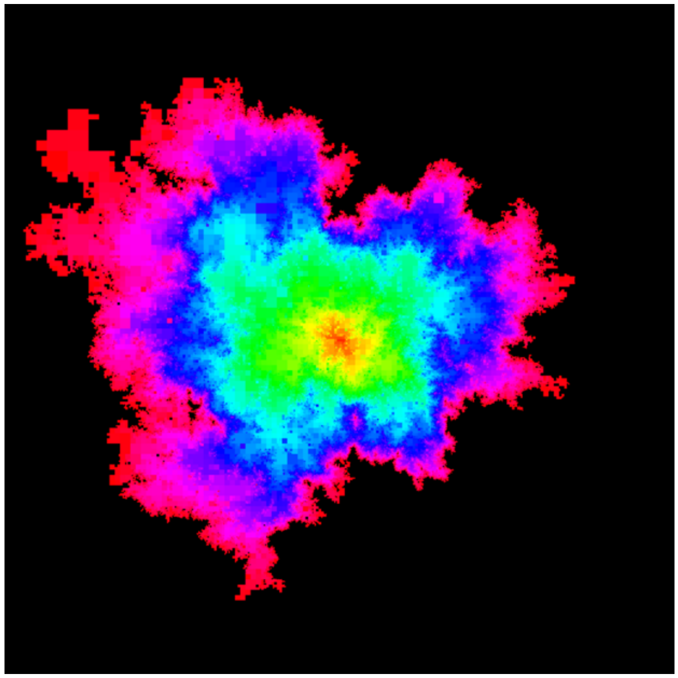

FIGURE 4.2. A metric ball (generated by QLE) on the plane.

by an analytical approach to the correlation functions. There is strong connection to the dimer models studied by Fisher, Kasteleyn, and Temperley.

4.14. **Self-avoiding walks.** Grimmett and Li have continued their work on the connective constants $\mu(G)$ of infinite, transitive graphs G. Amongst their results are the following.

They have analysed the relationship between the existence of height functions and the property of amenability, showing in particular that the Cayley graphs of elementary amenable groups have height functions, but that the Grigorchuk Cayley graph (which is amenable but not elementary amenable) does not. The Cayley graph of the Higman group (which is non-amenable) is another example of a graph without a height function. The identification of the Grigorchuk and Higman groups in this context resolves an earlier open question.

Let ϕ denote the golden ratio. They have explored the proposed inequality $\mu \geq \phi$, which may be valid for all infinite, transitive, cubic, simple graphs. This possible strengthening of their earlier inequality $\mu \geq \sqrt{2}$ has now been proved for certain families of cubic graphs, but it remains an open question in complete generality. For example, the best lower bound currently known for the Grigorchuk Cayley graph is of the form $\mu \geq \gamma$ for some known γ satisfying $\sqrt{2} < \gamma < \phi$.

5. Activities

5.1. **Outputs.** The following publications and preprints have been facilitated by funding through RaG. They are available via

http://www.statslab.cam.ac.uk/~grg/rag-pubs.html

Preprints from this report period

1. Liouville quantum gravity and the Brownian map II: geodesics and continuity of the embedding, Jason Miller, Scott Sheffield,
2. The work of Lucio Russo on percolation, Geoffrey Grimmett, Mathematics and Mechanics of Complex Systems
3. Universality of fluctuations in the dimer model, Nathanael Berestycki, Benoit Laslier, Gourab Ray
4. CLE percolations, Jason Miller, Scott Sheffield, Wendelin Werner
5. The half plane UIPT is recurrent, Omer Angel and Gourab Ray
6. The generalized master fields, Guillaume Cébron, Antoine Dahlqvist, Franck Gabriel
7. Universal constructions for spaces of traffics, Guillaume Cébron, Antoine Dahlqvist, Camille Male
8. Correlation inequalities for the Potts model, Geoffrey Grimmett, Mathematics and Mechanics of Complex Systems
9. Existence of self-accelerating fronts for a non-local reaction-diffusion equations, Nathanael Berestycki, Clément Mouhot, Gael Raoul
10. Bipolar orientations on planar maps and SLE_{12}, Richard Kenyon, Jason Miller, Scott Sheffield, David B. Wilson
11. Self-avoiding walks and amenability, Geoffrey Grimmett and Zhongyang Li
12. Critical surface of the hexagonal polygon model, Geoffrey Grimmett and Zhongyang Li, *J. Stat. Phys.* 163 (2016), 733–753.
13. The 1-2 model, Geoffrey Grimmett and Zhongyang Li. *Proc. 4th Ahlfors–Bers Symposium.*

Publications and preprints from previous report periods

1. Liouville quantum gravity and the Brownian map I: The $\mathrm{QLE}(8/3, 0)$ metric, Jason Miller, Scott Sheffield
2. An elementary approach to Gaussian multiplicative chaos, Nathanael Berestycki
3. Critical surface of the 1-2 model, Geoffrey Grimmett and Zhongyang Li
4. An axiomatic characterization of the Brownian map, Jason Miller, Scott Sheffield
5. Liouville quantum gravity spheres as matings of finite-diameter trees, Jason Miller, Scott Sheffield
6. Small-time fluctuations for the bridge of a sub-Riemannian diffusion, Ismael Bailleul, Laurent Mesnager, James Norris

7. Random walks on the random graph, Nathanael Berestycki, Eyal Lubetzky, Yuval Peres, Allan Sly
8. Near-critical spanning forests and renormalization, S. Benoist, L. Dumaz, W. Werner
9. Critical exponents on Fortuin–Kasteleyn weighted planar maps. N. Berestycki, B. Laslier, G. Ray
10. Conformal invariance of dimer heights on isoradial double graphs, Zhongyang Li, *Ann. de l'Instit. Henri Poincaré D*
11. Locality of connective constants, II. Cayley graphs, Geoffrey Grimmett and Zhongyang Li
12. Liouville quantum gravity and the Gaussian free field, Nathanael Berestycki, Scott Sheffield, Xin Sun
13. Cutoff for conjugacy-invariant random walks on the permutation group, Nathanael Berestycki, Bati Sengul
14. Locality of connective constants, I. Transitive graphs, Geoffrey Grimmett and Zhongyang Li
15. The Potts and random-cluster models, Geoffrey Grimmett
16. Measure solutions for the Smoluchowski coagulation–diffusion equation, James Norris
17. Cyclic polynomials in two variables, Catherine Bénéteau, Greg Knese, Lukasz Kosiáski, Constanze Liaw, Daniel Seco, Alan Sola, *Transactions of the AMS*
18. Surprise probabilities in Markov chains, James Norris, Yuval Peres, Alex Zhai
19. From Sine kernel to Poisson statistics, Romain Allez, Laure Dumaz, *Electronic J. Probab.* 19 (2014) 1–25
20. KPZ formula derived from Liouville heat kernel, N. Berestycki, C. Garban, R. Rhodes, V. Vargas.
21. A consistency estimate for Kac's model of elastic collisions in a dilute gas, J. Norris, *Adv. Appl. Probab.* 26 (2016), 102–108.
22. Random matrices in non-confining potentials, R. Allez, L. Dumaz, *J. Statist. Phys.* 160 (2015) 681–714
23. Tracy–Widom at high temperature, R. Allez, L. Dumaz, *J. Statist. Phys.* 156 (2014) 1146–1183.
24. Criticality, universality, and isoradiality, G. Grimmett, *Proc. 2014 ICM, Seoul.*
25. Cyclicity in Dirichlet-type spaces and extremal polynomials II: functions on the bidisk, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Pacific Journal of Mathematics* 276 (2015) 35–58
26. Small-particle limits in a regularized random Laplacian growth model, F. Johansson Viklund, A. Sola, A. Turner, *Commun. Math. Phys.* 334 (2015) 331–366
27. Discrete complex analysis and T-graphs, Z. Li.
28. Conformal invariance of isoradial dimers, Z. Li.

29. Coalescing Brownian flows: a new approach, N. Berestycki, C. Garban, A. Sen, *Ann. Prob.* (2015), 3177–3215.
30. Extendable self-avoiding walks, G. Grimmett, A. Holroyd, Y. Peres, *Ann. Inst. H. Poincaré D* 1 (2014) 61–75
31. Condensation of a two-dimensional random walk and the Wulff crystal, N. Berestycki, A. Yadin
32. The shape of multidimensional Brunet–Derrida particle systems, N. Berestycki, Lee Zhuo Zhao, *Ann. Appl. Prob.*
33. Counting self-avoiding walks, G. Grimmett, Z. Li
34. Percolation of finite clusters and infinite surfaces, G. Grimmett, A. Holroyd, G. Kozma, *Math. Proc. Cam. Phil. Soc. 156* (2014) 263–279
35. Diffusion in planar Liouville quantum gravity, N. Berestycki, *Ann. Inst. Henri Poincar Probab. Stat.* 51 (2015), 947–964.
36. Cyclicity in Dirichlet-type spaces and extremal polynomials, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Journal d'Analyse Mathématique* 126 (2015) 259–286
37. Expected discrepancy for zeros of random polynomials, I. Pritsker, A. Sola, *Proceedings of the American Mathematical Society* 142 (2014) 4251–4263
38. Elementary examples of Loewner chains generated by densities, A. Sola, *Annales Universitatis Mariae Curie-Sklodowska* A 67 (2013) 83–101.
39. Strict inequalities for connective constants of transitive graphs, G. Grimmett, Z. Li, *SIAM Journal of Discrete Mathematics* 28 (2014), 1306–1333
40. Diffusivity of a random walk on random walks, E. Boissard, S. Cohen, T. Espinasse, J. Norris, *Random Structures & Algorithms* 47 (2015), 267–283.
41. Uniqueness of infinite homogeneous clusters in 1–2 model, Z. Li, *Electron. Commun. Probab.* 19 (2014), Paper 23, 8 pp.
42. Bounds on connective constants of regular graphs, G. Grimmett, Z. Li, *Combinatorica* 35 (2015) 279–294.
43. Self-avoiding walks and the Fisher transformation, G. Grimmett, Z. Li, *European Journal of Combinatorics* 20 (2013), Paper P47, 14 pp.
44. Influence in product spaces, G. Grimmett, S. Janson, J. Norris, *Advances in Applied Probability* 48A (2016)
45. Critical branching Brownian motion with absorption: particle configurations, J. Berestycki, N. Berestycki, J. Schweinsberg, *Ann. Inst. Henri Poincar Probab. Stat.* 51 (2015), 1215–1250
46. Critical branching Brownian motion with absorption: survival probability, J. Berestycki, N. Berestycki, J. Schweinsberg, *Probab. Theory Related Fields* 160 (2014), 489–520.
47. Three theorems in discrete random geometry, G. Grimmett. *Probability Surveys* 8 (2011) 403–441

48. A small-time coupling between Lambda-coalescents and branching processes, J. Berestycki, N. Berestycki, V. Limic, *Annals of Applied Probability* 24 (2014) 449–475.
49. The genealogy of branching Brownian motion with absorption, J. Berestycki, N. Berestycki, J. Schweinsberg, *Annals of Probability* 41 (2013) 527–618
50. Percolation since Saint-Flour, G. Grimmett, H. Kesten, in *Percolation Theory at Saint-Flour*, Springer, 2012, pages ix–xxvii
51. Cycle structure of the interchange process and representation theory, N. Berestycki, G. Kozma, *Bull. Soc. Math. France* 143 (2015), 265–280.
52. Galton–Watson trees with vanishing martingale limit, N. Berestycki, N. Gantert, P. Moerters, N. Sidorova, *J. Stat. Phys.* 155 (2014) 737–762.
53. Critical temperature of periodic Ising models, Z. Li, *Communications in Mathematical Physics* 315 (2012) 337–381.
54. Spectral curve of periodic Fisher graphs, Z. Li, *Journal of Mathematical Physics* 55, 123301 (2014)
55. Bond percolation on isoradial graphs, G. Grimmett, I. Manolescu, *Probability Theory and Related Fields* 159 (2014) 273–327.
56. Asymptotic sampling formulae for Lambda-coalescents, J. Berestycki, N. Berestycki, V. Limic, *Ann. Inst. H. Poincaré B* 50 (2014), 715–731
57. 1–2 model, dimers, and clusters, Z. Li, *Electronic Journal of Probability* 19 (2014) Paper 48.
58. Large scale behaviour of the spatial Lambda–Fleming–Viot process, N. Berestycki, A. M. Etheridge, A. Veber, *Ann. Inst. H. Poincaré B* 49 (2013) 374–401
59. Hastings–Levitov aggregation in the small-particle limit, J. Norris, A. Turner, *Commun. Math. Phys.* (2012) 316, 809–841
60. Weak convergence of the localized disturbance flow to the coalescing Brownian flow, J. Norris, A. Turner, *Annals of Probability* 43 (2015) 935–970
61. Universality for bond percolation in two dimensions, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 3261–3283.
62. Inhomogeneous bond percolation on square, triangular, and hexagonal lattices, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 2990–3025.
63. Cluster detection in networks using percolation, G. Grimmett, E. Arias-Castro, *Bernoulli* 19 (2013) 676–719

5.2. **Seminars.** The weekly probability seminar has been lively as always. Details of events may be found at

http://talks.cam.ac.uk/show/archive/9938.

5.3. **Visitors.** Cambridge Probability has received a number of visitors in 2015–16, for short and longer periods, including the many participants at the INI programme described above. The following visitors are connected directly to RaG.

- Rick Kenyon, Nov 2015
- Yuval Peres, Jan 2016
- Russell Lyons, Feb 2016
- Juhan Aru, Feb 2016
- Tom Hutchcroft, Mar 2016
- Julien Dubédat, May 2016
- Wendelin Werner, Feb 2016 (and other visits)

5.4. **Visits by members of RaG.** Members of RaG have made numerous visits to other institutions, and have participated in numerous conferences and workshops. Listed here are visits made by *research fellows only*.

5.5. **Scientific visits.**

- Aug 2015: Helsinki [Laslier].
- Dec 2015: Grenoble [Laslier].
- Nov 2015: ETH Zurich visit [Ray].
- Apr 2016: Geneva visit [Ray].
- Jun 2016: Research visit to Lancaster [Dahlqvist].
- Jun 2016: Lyon [Laslier].
- May 2016: Research visit to Paris 6 University [Dahlqvist].
- Dec 2015: Research visit to Helsinki [Dahlqvist].

5.6. **Conferences.**

- May 2016: Dimer day, Cambridge [Laslier, Ray, Dahlqvist].
- Jun 2016: Les journées scientifiques de Nantes [Laslier].
- Jun 2016: Rencontres de mcanique statistique, Creteil [Laslier].
- Apr 2016: UK Easter probability meeting, Lancaster [Laslier, Ray].
- Jun 2016: Arbres et cartes aléatoires : aspects probabilistes et combinatoires, Marseille [Laslier].
- Jun 2016:, Transversal aspects of tilings, Iles d'Oleron [Laslier].
- Mar 2016. Bristol BMC conference [Ray].
- Apr 2016: Geometric models in probability workshop, Darmstadt [Ray].
- Jul 2016: World Congress in Probability and Statistics, Toronto [Ray].
- May 2016: Notions of freeness, Saarbrücken [Dahlqvist].

6. Future activities

Amongst our immediate targets are the following.

- Planning for the next day of industrial outreach.
- Planning the RaG closing workshop, scheduled for 25–30 June 2017.

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