NEW FRONTIERS IN RANDOM GEOMETRY (RaG)
EP/I03372X/1
REPORT 1/7/16 – 30/8/17

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1. Management Process

The Management Committee (MC) comprises the three investigators and the four members of the external Advisory Board (AB), namely Yuval Peres, Stanislav Smirnov, Balint Tóth and Wendelin Werner. The local managers have met weekly during term, and more formally about every two months. The advice of the AB has been sought on a variety of matters including the hiring process. One member of the AB (Werner) spent several periods in Cambridge during the period of this report. A meeting of the Advisory Board was held on 30 June 2017, and the minutes are attached in Section 6.

2. Personnel

Three postdoctoral research fellows have left the team since August 2016:

- Benoît Laslier[1], employed from 1 September 2014 to 31 August 2016,
- Gourab Ray[2], employed from 1 September 2014 to 30 June 2017,
- Antoine Dahlqvist[3], employed from 1 October 2015 to 31 August 2017,
- Marcin Lis[4], employed from 1 October 2016 to 31 August 2017.

3. Scientific meetings

In addition to the weekly seminar[5], members of the group organised the closing workshop of RaG entitled RaGeCam[6], from 25–30 June 2017. Every current and past research associate of RaG was invited to present their work in a lecture, and in addition a number of internationally prominent mathematicians accepted invitations to contribute. The list of speakers was: Kenyon, Ray, Russkikh, Lis, Cimasoni, Li, Laslier, Miller, Kassel, Sheffield,

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Date: October 17, 2017.

http://www.statslab.cam.ac.uk/~grg/rag.html.

[1] http://www.lpma-paris.fr/pageperso/laslier/

[2] https://www.uvic.ca/science/math-statistics/people/home/faculty/ray_gourab.php

[3] http://www.statslab.cam.ac.uk/~ad814/

[4] https://www.dpmms.cam.ac.uk/people/ml814/

[5] http://talks.cam.ac.uk/show/index/9938

[6] http://www.statslab.cam.ac.uk/~grg/rage.html

Qian, Levy, Dahlqvist, Sola, Silvestri. Support was offered to a number of PhD students and postdocs from around the UK.

4. Research Programme (selected highlights)

Some highlights of the year's research are as follows.

4.1. **Universality of dimer models.** Berestycki, Laslier and Ray have continued their investigation of the fluctuations of the dimer model. They prove that, on for a Temperleyan graph embedded on a Riemann surface, an invariance principle for random walk on the graph implies that the fluctuations of the height function are universal and conformally invariant. They have previously proved such a result for a simply connected domain.

To be more precise, the height function on a topological surface is decomposed into two parts, a *scalar* component and a topological '*instanton*' component. They have proved that the components converge jointly. The current proof is complete for the case of Euler characteristic 0 (torus and annulus), and in general the proof is complete assuming a certain convergence result which they plan to prove in an upcoming paper. The proof relies on proving the convergence of cycle rooted spanning forests (CRSF) on a surface and then establishing that such a limiting object is universal and conformally invariant. This extends Lawler, Schramm and Werner's result of convergence of Uniform Spanning Tree on a simply connected domain. In particular, the joint convergence of the CRSF and the field is established and it is also proved that the limiting field measurable with respect to the limiting CRSF.

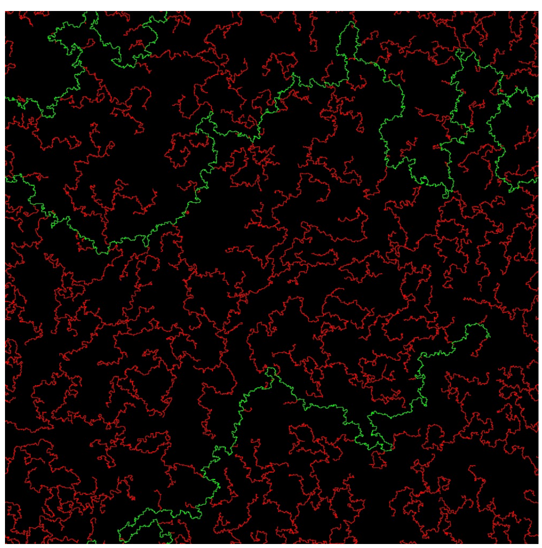

Figure 4.1. Sample of a CRSF on a 1000 by 1000 torus with 500 branches. It has one non-contractible branch shown in green

Berestycki, Laslier and Ray have continued their work on the fluctuations of the dimer model with non-flat boundary condition for any graph satisfying the invariance principle. Their previous work proves the convergence for the flat case, in fact they prove a central limit theorem in a certain associated graph called the *T-graph* implies universality. In this work this central limit theorem is proved, thereby completing the picture for a non-flat domain.

4.2. **Random homomorphisms.** Duminil-Copin, Harel, Laslier, Raoufi and Ray have investigated random homomorphisms from $\mathbb{Z}^2$ to $\mathbb{Z}$ and, more generally, the 6-vertex model with parameter $c \geq 1$. The 6 vertex model is associated with the dimer model for $c = 0$ and with the FK model for $c \geq \sqrt{2}$. The goal is to prove a dichotomy for Russo–Seymour–Welsh type observables. As observables they consider certain *crossing events*. A dichotomy result would be of the nature that such a crossing would either have positive probability or would decay exponentially as the mesh size vanishes.

4.3. **Characterization of the GFF.** Berestycki, Powell and Ray have worked on a characterization result for the Gaussian free field. They have investigated properties of a random distribution which is conformally invariant and has a certain domain Markov property. They show that, under extra mild assumptions, such a distribution must be a Gaussian free field.

4.4. **Rohde–Schramm theorem, via the Gaussian Free Field.** The Rohde–Schramm theorem is the foundational result in SLE theory, and asserts that SLE exists as a random curve, almost surely (that is, the solution of the Loewner equation driven by Brownian motion is generated by a curve almost surely). The proof of this result is peculiar: the SLE estimates of Rohde and Schramm are valid for all values κ except $\kappa = 8$, in which case the result is only known as a consequence of convergence of Loop-Erased Random Walk (LERW) to SLE_2 by Lawler, Schramm and Werner, together with Wilson's algorithm which relates the Uniform Spanning Tree (UST) to LERW. Since the UST is obviously generated by a curve, so must SLE_8 in the limit! It has been an open question since that paper to devise a unified 'continuous' proof which requires no special 'discrete' arguments for $\kappa = 8$.

Berestycki and Jackson obtained a proof of the Rohde–Schramm theorem that uses a coupling to an underlying Gaussian Free Field (the "reverse" coupling of Liouville quantum gravity) to establish properties of the SLE through GFF estimates. At the moment, the work yields an alternative proof of the Rohde–Schramm theorem for exactly the same cases as the original proof of Rohde and Schramm (namely, all values of κ except $\kappa = 8$).

4.5. **Random GUE matrices and Gaussian multiplicative chaos.** Links between GUE random matrices and random planar maps (or Liouville quantum gravity) are well known to the mathematical physics community, at least since the work of Itzykhson. In parallel, it has been known for some time that the logarithm of the characteristic polynomial of GUE random matrices converges to a log-correlated field in 1 dimension, which can be thought of as the restriction to the real line of the Neumann GFF in the half-plane.

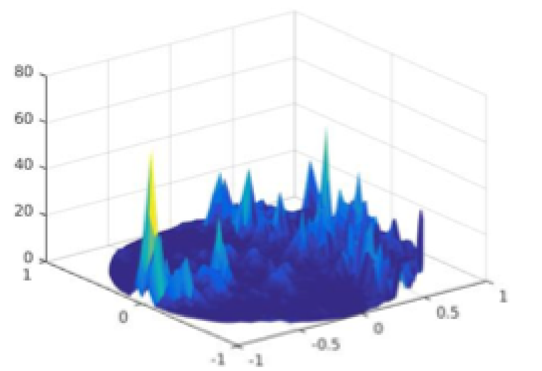

FIGURE 4.2. The absolute value of the (normalised) characteristic polynomial for a random matrix taken from the Ginibre ensemble, $N = 1200$. It is believed this converges to Gaussian multiplicative chaos (also known as Liouville quantum gravity). The analogous statement for the GUE was obtained by Berestycki, Webb and Wong.

Work by Berestycki, Webb and Wong has showed a strengthening of this result in the sense of exponentiation. This relates the characteristic polynomial to the random measures constructed by the theory of Gaussian multiplicative chaos (i.e., the boundary Liouville quantum gravity measure). Mathematically, the proof relies on an analysis of a Riemann–Hilbert problem and the method of steepest descent of Deift and Zhou.

4.6. **A conjecture of Bolthausen.** Bolthausen conjectured in 1994 that a random walk trajectory, penalized by $\exp(-|R_n|)$ where R_n is the range of the walk, is localized on a Euclidean ball of radius $\rho_d n^{1/(d+2)}$ where ρ_d is a certain deterministic constant. Bolthausen proved this result for the case $d = 2$ while the case $d \geq 3$ remains open.

Work is in progress by Berestycki and Cerf on this question. At the moment they have a limit shape theorem in the sense of L^1 distance of the local time of the walk. A key step is to use recent discrete Faber–Krahn inequalities in dimension $d \geq 3$. One highly nontrivial technical issue is that such inequalities are only known in the full plane and not in the torus. This makes it impossible to project random walk on the torus and make use of the Donsker–Varadhan theory (which requires a compact state space). Hence a byproduct of the analysis is an extension of this theory to all of $\mathbb{Z}^d$, which should have independent applications.

Additional difficulties remain in order to obtain the shape theorem in the Hausdorff sense, where work continues.

4.7. **Mixing times on random graphs with heavy tails.** Berestycki, Lubetzky, Peres and Sly have shown that, on an Erdős–Renyi random graph $G(n, p)$ with $p = \lambda/n$ and $\lambda > 1$ (so there is a giant component), a random walk from a typical vertex undergoes the cutoff phenomenon with a mixing time of $C(\lambda) \log n$ where $C(\lambda)$ is a certain constant which depends only on the average degree. This result is extended to random graphs from the configuration model provided that the maximum degree is not too large; if the minimum degree is at least 3 then the cutoff holds uniformly in the starting point, and the same holds in the easier case of the nonbacktracking random walk.

This raises the question of random graphs with a heavy-tailed degree distribution, and work has begun on this by Berestycki, van der Hofstad and Salez. They hope to show that, starting from a uniform point, the presence of heavy tails does not affect the mixing time or the cutoff phenomenon. However, if the minimum degree is at least three, depending on the starting point the cutoff phenomenon may or may not happen. In particular there are unusual starting points such that the total variation distance to equilibrium evolves with a finite number of macroscopic jumps. The number of finite jumps can be as large as desired.

4.8. **Yang–Mills measure and master field on the sphere.** Dahlqvist and Norris have completed their work on the convergence of the Wilson loops towards a master field for the Yang–Mills measure on the sphere, with large unitary matrices as a structure group, solving a conjecture of I. M. Singer. In ongoing work, Dahlqvist is investigating extensions for other topologies: considering random connections over non-trivial fiber bundles and loops on different compact Riemann surfaces without boundary. In the case of the sphere, the topology of the fibers relates to the winding of self-avoiding particles on the circle. It appears that the latter can be expressed in terms of the barycenter of a discrete one-dimensional Coulomb gas. These expressions complete the duality formulas discovered in the former work with Norris.

4.9. **Large deviations for the Yang–Mills measure.** Dahlqvist has investigated another approach to the above work with Norris, aiming at showing a large deviation principle for the convergences they considered. A preliminary result is to prove a large deviation principle for the spectrum of a Brownian motion on large unitary groups. This work allows us to study the empirical distributions of Brownian bridges on these groups with different boundary conditions, expressing their limit in terms of Euler equations for compressible fluids on the unit circle with negative pressure. This approach makes rigorous some of the arguments of the physicists Gross and Matytsin. Another aim of this study is to understand the so-called Douglas–Kazakov phase transition phenomenons displayed by these models. A preliminary result gives an elementary proof for the subcritical regime of the unitary Brownian loop, where its eigenvalues have no winding.

4.10. **Double random currents.** The random current model is a dependent percolation model derived from the power series expansion of the Ising model partition function. A *double* random current model is simply a superimposition of two i.i.d. random currents. Its special combinatorial feature in the form of the *switching lemma* has been employed many times in recent years to prove finer properties of the Ising model in $d \geq 2$ dimensions.

For $d = 2$, a fact that is intrinsically related to the switching lemma is that the double random current model is equivalent to a dimer model on a modified graph. Lis and Duminil-Copin have used this property to prove the vanishing of the magnetization in the critical Ising model on any planar biperiodic graph.

Duminil-Copin, Lis and Ray are aiming to prove conformal invariance of the critical double random current in the scaling limit.

4.11. **Loop soup winding fields.** A random walk loop soup is a Poisson collection of random walk trajectories conditioned to come back to their starting point. It is a celebrated result that the continuous analog — the Brownian loop soup of Scheffield and Werner — encodes the scaling limits of planar critical models of statistical mechanics.

Lis, Camia and van de Brug have studied the winding fields of loop soups, i.e. random functions of the form $V(z) = e^{i\beta N_z}$ where β is an angle parameter and N_z is the total winding number about z of all loops in the soup. In the special case $\beta = \pi$, the field becomes a ± 1 *spin field*. They prove several facts about the convergence of the discrete spin field to a conformally invariant random generalized function. They relate the field to known models of statistical mechanics including the loop-erased random walk, the discrete Gaussian free field and the Ising model. Challenging open questions include establishing the scaling limit of the contours between clusters of $+1$ and -1 spins.

4.12. **Ising preholomorphic observables on decorated lattices.** Smirnov and Chelkak–Smirnov have identified and proved convergence of discrete holomorphic observables in the critical planar Ising model on isoradial graphs. An intriguing question is whether the class of isoradial graphs is the largest for which one can obtain such results. In ongoing work, Lis has identified and partially studied preholomorphic observables on decorated regular lattices (this class includes for example the critical square/octagon lattices which cannot be isoradially embedded in the plane).

4.13. **Self-avoiding walks.** In their ongoing project concerning self-avoiding walks (SAWs), Grimmett and Li have studied the validity of the inequality $\mu \geq \phi$ for the connective constant μ of an infinite, transitive, cubic graph. They have proved this for a number of classes of such graphs, including topologically locally finite graphs, and Cayley graphs of certain families of groups including 2-ended groups. Their work has highlighted a number of beautiful open problems concerning Cayley graphs, and more generally transitive graphs.

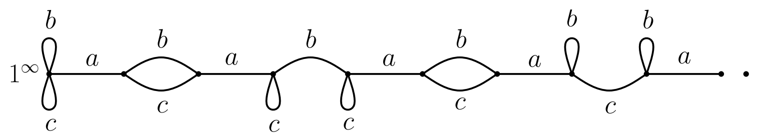

FIGURE 4.3. The one-ended orbital Schreier graph of the ray 1^∞ under the Grigorchuk group.

4.14. **Locality for percolation.** The locality problem is to decide when the critical point of a given model is a continuous function of the underlying graph. Earlier work of Grimmett and Li has been directed at this question in the case of self-avoiding walks. More recently, Grimmett and Minter have considered the corresponding percolation problem, and have partial observations concerning transitive graphs that extend some of the previous results of Martineau and Tassion for Cayley graphs of abelian groups.

4.15. **Bulk scaling limit and fluctuations for Hastings–Levitov aggregation.** Norris, Silvestri and Turner are investigating the planar aggregation model HL(α) with parameter $\alpha \in [0,1]$. This follows on from work of Norris and Turner for $\alpha = 0$ and later Silvestri, who determined the fluctuations for $\alpha = 0$. It also seeks to extend work of Sola, Turner and Viklund for positive α. The case $\alpha = 1$ may be seen as an off-lattice variant of the Eden model. A promising new approach deals directly with the 'forward' conformal mappings, which encode the clusters, rather than the mapping-out functions previously used. This opens the possibility to use infinite-dimensional Markovian limits to handle the case $\alpha > 0$. Initial results suggest a ball theorem for all $\alpha \leq 1$, with α-dependent Gaussian fluctuations for $\alpha < 1$. The analysis of fluctuations when $\alpha = 1$, which appears as a critical case, is more challenging and is the subject of ongoing work.

5. Activities

5.1. **Outputs.** The following publications and preprints have been facilitated by funding through RaG. They are available via

http://www.statslab.cam.ac.uk/~grg/rag-pubs.html

Preprints from this report period

1. Yang–Mills measure and the master field on the sphere, Antoine Dahlqvist, James Norris.
2. The parabolic Anderson model on Riemann surfaces, Antoine Dahlqvist, Joscha Diehl and Bruce Driver.
3. The planar Ising model and total positivity, Marcin Lis, *J. Statist. Phys.* 166 (2017) 72–89.

4. Non-backtracking loop soups and statistical mechanics on spin networks, Federico Camia and Marcin Lis, *Annales Henri Poincaré* 18 (2017), 403–433.
5. Probability on Graphs, Geoffrey Grimmett, Cambridge University Press, 2nd edition, 2018, in press.
6. Self-avoiding walks and connective constants, Geoffrey Grimmett, Zhongyang Li.
7. The Rohde–Schramm theorem, via the Gaussian free field, Nathanael Berestycki, Henry Jackson.
8. Lozenge tiling dynamics and convergence to the hydrodynamic equation, B. Laslier, F. L. Toninelli.
9. A unimodular Liouville hyperbolic souvlaki — an appendix to arXiv:1603.06712, Gabor Pete, Gourab Ray.
10. Hyperbolic and parabolic unimodular random maps, Omer Angel, Tom Hutchcroft, Asaf Nachmias, Gourab Ray.
11. Classification of scaling limits of uniform quadrangulations with a boundary, Erich Baur, Gregory Miermont, Gourab Ray.
12. Cubic graphs and the golden mean, Geoffrey Grimmett, Zhongyang Li.
13. Random Hermitian matrices and Gaussian multiplicative chaos, N. Berestycki, C. Webb, M.-D. Wong, *Probab. Th. Related Fields.*
14. A note on dimers and T-graphs, Nathanael Berestycki, Benoit Laslier, Gourab Ray .
15. Liouville quantum gravity and the Brownian map III: the conformal structure is determined, Jason Miller, Scott Sheffield.
16. Hydrodynamic limit equation for a lozenge tiling Glauber dynamics, B. Laslier, F. L. Toninelli.

Publications and preprints from previous report periods

1. Liouville quantum gravity and the Brownian map II: geodesics and continuity of the embedding, Jason Miller, Scott Sheffield.
2. The work of Lucio Russo on percolation, Geoffrey Grimmett, *Mathematics and Mechanics of Complex Systems* 4 (2016) 199–211.
3. Universality of fluctuations in the dimer model, Nathanael Berestycki, Benoit Laslier, Gourab Ray.
4. CLE percolations, Jason Miller, Scott Sheffield, Wendelin Werner.
5. The half plane UIPT is recurrent, Omer Angel and Gourab Ray.
6. The generalized master fields, Guillaume Cébron, Antoine Dahlqvist, Franck Gabriel.
7. Universal constructions for spaces of traffics, Guillaume Cébron, Antoine Dahlqvist, Camille Male.
8. Correlation inequalities for the Potts model, Geoffrey Grimmett, *Mathematics and Mechanics of Complex Systems* 4 (2016) 327–334.
9. Existence of self-accelerating fronts for a non-local reaction-diffusion equations, Nathanael Berestycki, Clément Mouhot, Gael Raoul.

10. Bipolar orientations on planar maps and SLE_{12}, Richard Kenyon, Jason Miller, Scott Sheffield, David B. Wilson.
11. Self-avoiding walks and amenability, Geoffrey Grimmett and Zhongyang Li.
12. Critical surface of the hexagonal polygon model, Geoffrey Grimmett and Zhongyang Li, *J. Statist. Phys.* 163 (2016), 733–753.
13. The 1-2 model, Geoffrey Grimmett and Zhongyang Li. *In the Tradition of Ahlfors–Bers, VII*, Proceedings of the Sixth Ahlfors–Bers Colloquium, 2014 *Contemporary Mathematics* 696 (2017) 130–152.
14. Liouville quantum gravity and the Brownian map I: The $\mathrm{QLE}(8/3, 0)$ metric, Jason Miller, Scott Sheffield.
15. An elementary approach to Gaussian multiplicative chaos, Nathanael Berestycki, *Electronic Communications in Probability.*
16. Critical surface of the 1-2 model, Geoffrey Grimmett and Zhongyang Li, *International Mathematics Research Notices* (2017) .
17. An axiomatic characterization of the Brownian map, Jason Miller, Scott Sheffield.
18. Liouville quantum gravity spheres as matings of finite-diameter trees, Jason Miller, Scott Sheffield.
19. Small-time fluctuations for the bridge of a sub-Riemannian diffusion, Ismael Bailleul, Laurent Mesnager, James Norris.
20. Random walks on the random graph, Nathanael Berestycki, Eyal Lubetzky, Yuval Peres, Allan Sly, *Annals of Probability.*
21. Near-critical spanning forests and renormalization, S. Benoist, L. Dumaz, W. Werner.
22. Critical exponents on Fortuin–Kasteleyn weighted planar maps. N. Berestycki, B. Laslier, G. Ray, *Commun. Math. Phys.*.
23. Conformal invariance of dimer heights on isoradial double graphs, Zhongyang Li, *Ann. de l'Instit. Henri Poincaré D.*
24. Connective constants and height functions of Cayley graphs, Geoffrey Grimmett and Zhongyang Li, *Transactions of the AMS* 369 (2017) 5961–5980.
25. Liouville quantum gravity and the Gaussian free field, Nathanael Berestycki, Scott Sheffield, Xin Sun.
26. Cutoff for conjugacy-invariant random walks on the permutation group, Nathanael Berestycki, Bati Sengul.
27. Locality of connective constants, Geoffrey Grimmett and Zhongyang Li.
28. The Potts and random-cluster models, Geoffrey Grimmett.
29. Measure solutions for the Smoluchowski coagulation–diffusion equation, James Norris.
30. Cyclic polynomials in two variables, Catherine Bénéteau, Greg Knese, Lukasz Kosiáski, Constanze Liaw, Daniel Seco, Alan Sola, *Transactions of the AMS.*

31. Surprise probabilities in Markov chains, James Norris, Yuval Peres, Alex Zhai.
32. From Sine kernel to Poisson statistics, Romain Allez, Laure Dumaz, *Electronic J. Probab.* 19 (2014) 1–25.
33. KPZ formula derived from Liouville heat kernel, N. Berestycki, C. Garban, R. Rhodes, V. Vargas, *Journal of the London Mathemetical Society*.
34. A consistency estimate for Kac's model of elastic collisions in a dilute gas, J. Norris, *Adv. Appl. Probab.* 26 (2016), 102–108.
35. Random matrices in non-confining potentials, R. Allez, L. Dumaz, *J. Statist. Phys.* 160 (2015) 681–714
36. Tracy–Widom at high temperature, R. Allez, L. Dumaz, *J. Statist. Phys.* 156 (2014) 1146–1183.
37. Criticality, universality, and isoradiality, G. Grimmett, *Proc. 2014 ICM, Seoul*, vol. IV, 25–48.
38. Cyclicity in Dirichlet-type spaces and extremal polynomials II: functions on the bidisk, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Pacific Journal of Mathematics* 276 (2015) 35–58 .
39. Small-particle limits in a regularized random Laplacian growth model, F. Johansson Viklund, A. Sola, A. Turner, *Commun. Math. Phys.* 334 (2015) 331–366.
40. Discrete complex analysis and T-graphs, Z. Li, preprint, 2014.
41. Conformal invariance of isoradial dimers, Z. Li, https://arxiv.org/abs/1309.0151.
42. Coalescing Brownian flows: a new approach, N. Berestycki, C. Garban, A. Sen, *Ann. Prob.* (2015), 3177–3215.
43. Extendable self-avoiding walks, G. Grimmett, A. Holroyd, Y. Peres, *Ann. Inst. H. Poincaré D* 1 (2014) 61–75.
44. Condensation of a two-dimensional random walk and the Wulff crystal, N. Berestycki, A. Yadin.
45. The shape of multidimensional Brunet–Derrida particle systems, N. Berestycki, Lee Zhuo Zhao, *Ann. Appl. Prob.*
46. Counting self-avoiding walks, G. Grimmett, Z. Li, Accepted in Tsinghua Lectures in Mathematics, 2017.
47. Percolation of finite clusters and infinite surfaces, G. Grimmett, A. Holroyd, G. Kozma, *Math. Proc. Cam. Phil. Soc. 156* (2014) 263–279.
48. Diffusion in planar Liouville quantum gravity, N. Berestycki, *Ann. Inst. Henri Poincaré Probab. Stat.* 51 (2015), 947–964.
49. Cyclicity in Dirichlet-type spaces and extremal polynomials, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Journal d'Analyse Mathématique* 126 (2015) 259–286.
50. Expected discrepancy for zeros of random polynomials, I. Pritsker, A. Sola, *Proceedings of the American Mathematical Society* 142 (2014) 4251–4263.

51. Elementary examples of Loewner chains generated by densities, A. Sola, *Annales Universitatis Mariae Curie-Sklodowska* A 67 (2013) 83–101.
52. Strict inequalities for connective constants of transitive graphs, G. Grimmett, Z. Li, *SIAM Journal of Discrete Mathematics* 28 (2014), 1306–1333.
53. Diffusivity of a random walk on random walks, E. Boissard, S. Cohen, T. Espinasse, J. Norris, *Random Structures & Algorithms* 47 (2015), 267–283.
54. Uniqueness of infinite homogeneous clusters in 1–2 model, Z. Li, *Electron. Commun. Probab.* 19 (2014), Paper 23, 8 pp.
55. Bounds on connective constants of regular graphs, G. Grimmett, Z. Li, *Combinatorica* 35 (2015) 279–294.
56. Self-avoiding walks and the Fisher transformation, G. Grimmett, Z. Li, *European Journal of Combinatorics* 20 (2013), Paper P47, 14 pp.
57. Influence in product spaces, G. Grimmett, S. Janson, J. Norris, *Advances in Applied Probability* 48A (2016) 145–152.
58. Critical branching Brownian motion with absorption: particle configurations, J. Berestycki, N. Berestycki, J. Schweinsberg, *Ann. Inst. Henri Poincaré Probab. Stat.* 51 (2015), 1215–1250.
59. Critical branching Brownian motion with absorption: survival probability, J. Berestycki, N. Berestycki, J. Schweinsberg, *Probab. Theory Related Fields* 160 (2014), 489–520.
60. Three theorems in discrete random geometry, G. Grimmett. *Probability Surveys* 8 (2011) 403–441.
61. A small-time coupling between Lambda-coalescents and branching processes, J. Berestycki, N. Berestycki, V. Limic, *Annals of Applied Probability* 24 (2014) 449–475.
62. The genealogy of branching Brownian motion with absorption, J. Berestycki, N. Berestycki, J. Schweinsberg, *Annals of Probability* 41 (2013) 527–618.
63. Percolation since Saint-Flour, G. Grimmett, H. Kesten, in *Percolation Theory at Saint-Flour*, Springer, 2012, pages ix–xxvii.
64. Cycle structure of the interchange process and representation theory, N. Berestycki, G. Kozma, *Bull. Soc. Math. France* 143 (2015), 265–280.
65. Galton–Watson trees with vanishing martingale limit, N. Berestycki, N. Gantert, P. Moerters, N. Sidorova, *J. Statist. Phys.* 155 (2014) 737–762.
66. Critical temperature of periodic Ising models, Z. Li, *Communications in Mathematical Physics* 315 (2012) 337–381.
67. Spectral curve of periodic Fisher graphs, Z. Li, *Journal of Mathematical Physics* 55, 123301 (2014).
68. Bond percolation on isoradial graphs, G. Grimmett, I. Manolescu, *Probability Theory and Related Fields* 159 (2014) 273–327.

69. Asymptotic sampling formulae for Lambda-coalescents, J. Berestycki, N. Berestycki, V. Limic, *Ann. Inst. H. Poincaré B* 50 (2014), 715–731.
70. 1–2 model, dimers, and clusters, Z. Li, *Electronic Journal of Probability* 19 (2014) Paper 48.
71. Large scale behaviour of the spatial Lambda–Fleming–Viot process, N. Berestycki, A. M. Etheridge, A. Veber, *Ann. Inst. H. Poincaré B* 49 (2013) 374–401.
72. Hastings–Levitov aggregation in the small-particle limit, J. Norris, A. Turner, *Commun. Math. Phys.* (2012) 316, 809–841.
73. Weak convergence of the localized disturbance flow to the coalescing Brownian flow, J. Norris, A. Turner, *Annals of Probability* 43 (2015) 935–970.
74. Universality for bond percolation in two dimensions, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 3261–3283.
75. Inhomogeneous bond percolation on square, triangular, and hexagonal lattices, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 2990–3025.
76. Cluster detection in networks using percolation, G. Grimmett, E. Arias-Castro, *Bernoulli* 19 (2013) 676–719.

5.2. **Seminars.** The weekly probability seminar has been lively as always. Details of events may be found at

http://talks.cam.ac.uk/show/archive/9938.

5.3. **Visitors.** Cambridge Probability has received a number of visitors in 2016–17, for short and longer periods. The following visitors are connected directly to RaG.

- Matan Harel, October 2016
- Titus Lupu, October 2016
- Persi Diaconis, May 2016
- Aran Raoufi, May 2016
- Rick Kenyon, June 2016
- David Cimasoni, June 2016
- Adrien Kassel, June 2016
- Scott Sheffield, June 2016
- Thierry Lévy, June 2016
- Wendelin Werner, several visits

5.4. **Visits by members of RaG.** Members of RaG have made numerous visits to other institutions, and have participated in numerous conferences and workshops. Listed here are visits made by *research fellows only*.

5.5. **Scientific visits.**

- Nov 2016: IHES, Paris [Ray]
- Nov 2016: Portsmouth [Lis]

- Dec 2016: Warwick [Dahlqvist]
- Feb 2017: Leipzig [Dahlqvist]
- Feb 2017: Abu Dhabi [Lis]
- Apr 2017: Wrocław [Lis]
- May 2017: Utrecht [Lis]
- Jun 2017: Toulouse [Dahlqvist]
- Jun 2017: Amsterdam [Lis]
- Jun 2017: Cambridge [all RaG personnel]

5.6. Conferences.

- Jan 2017: Marseille [Ray]
- Jan 2017: IHP, Paris [Ray]
- Jan 2017: Marseille [Lis]
- Jan 2017: Paris [Lis]
- Jan 2017: Paris, IHP, workshop [Dahlqvist]
- Feb 2017: Paris, IHP, workshop [Dahlqvist]
- Jun 2017: Lyon, Random matrices [Dahlqvist]

6. Minutes of the RaG Advisory Board meeting, 30 June 2017, 4pm

Present: *Investigators*: Nathanael Berestycki (NB), Geoffrey Grimmett (GG, chair), James Norris (JN), *Advisory Board*: Balint Tóth (BT), Wendelin Werner (WW), joined by telephone by Yuval Peres (YP), Stanislav Smirnov (SS), *EPSRC*: Jan Taylor.

1. *Minutes and matters arising.*

   The minutes and past annual reports were received and approved.
2. *Programme review.*

   GG thanked everyone for making time to participate, and he summarized the current position. The RaG grant end-date was 31 August 2017, so this was the final AB meeting; RaG had employed 7 very talented postdocs, of whom 1 was staying in Cambridge, and 6 have proceeded (or are to proceed) to positions in other universities around Europe and North America/Canada. There had been three scientific meetings supported by RaG, and partial support for a six-month programme at the Isaac Newton Institute. The scientific achievements had been strongly supported by an active visitor programme which had enabled numerous leading individuals to visit Cambridge, and a number of visits by members of RaG to other institutions. Around 100 items of output have been supported by the grant.
3. *Summary of appointments.*

   No new appointments have been made this year. Marcin Lis will continue in Cambridge for a further year with funding from another source.

4. *Review of scientific programme.*

The reports from previous years up to 2016 were tabled, and GG, JN, NB presented verbal reports on activities so far during the last year. The final report would summarize the achievements of the entire programme since 2011, and would be prepared in the autumn and circulated to Board members.

External members of the Board were invited to comment on the progress achieved by RaG. WW said the entire programme had been a huge success, and had been transformative for the scientific field and for Cambridge's role internationally. The Cambridge Stats Lab, already a great Lab, had been able to develop further its identity. The presence of distinguished postdocs had created dynamism and vibrancy. Overall, the funds had been very well spent.

SS was very positive about the programme. It was in an interesting and important area of probability and mathematical physics, combining ideas from different fields. One novel aspect had been the use of results in the continuum to understand discrete models.

YP echoed the remarks of WW and SS. This had been a dramatically successful programme with an astounding breadth of achievement. The scientific area had developed greatly as a result, and there had been significant interaction with other leading groups internationally, including Paris, Geneva, and Zurich. RaG had been a major magnet that had attracted leaders to Cambridge. YP remarked that the notable successes in some problem areas had inevitably distracted attention from others.

BT expressed his full agreement with the previous remarks. He noted the consistency of the programme in its scientific level and directions, supported by its variations into related themes. There had been three effective scientific meetings. RaG was one of the most important programmes worldwide in probability over the last 5 years.

There followed a discussion of future steps to consolidate and develop the progress achieved in RaG. JT asked of plans to build upon staff and relationships developed via RaG. He said that probability and statistics were at the heart of EPSRC funding, and that applied probability remains a key area for the Council.

GG invited thoughts on how to underpin longer-term impact and contribution to UK science beyond the end of the RaG programme.

WW emphasized the core success of RaG in supporting the best young people, including PhD students and postdocs. The areas covered by RaG have attracted many of the best younger researchers in mathematics from around the world. It is important that such brilliant individuals can be supported by EPSRC funds. In response to a question from JT, GG explained that all the funds of RaG had supported science, and none had been expended on administrative or equipment support.

JT mentioned the significance of impact within the current funding model of the Research Councils. WW said that, while the topics of RaG have many connections to application areas (such as physics, biology, and network science), the principal target of pure mathematics is to develop conceptual advances rather than to strive for direct influence on applications. JT proposed that potential connections to other sciences receive attention in the final report.

5. *Workshops review.*

   It was noted that there had been 3 workshops plus a six-month Newton Institute programme. The final workshop had taken place over the week of the final AB meeting, and had featured talks by 6 of the 7 postdocs employed within RaG (with one exception owing to family responsiblity), together with a small number of distinguished scientists working in related fields.

6. *Any other business.*

   There was none.

GG thanked the Board members again for their contributions over the lifetime of RaG. The meeting closed at 5pm.

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