

## Percolation and related topics I

Exercises will be taken mostly from the second edition of my book *Probability on Graphs*, published by Cambridge University Press in 2018, and available online at

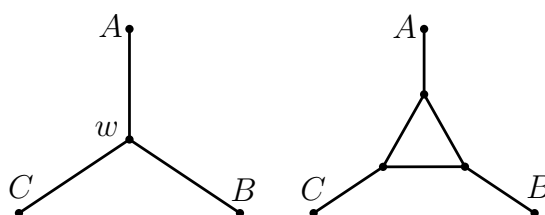
<http://www.statslab.cam.ac.uk/~grg/teaching/perc19.html>

This set of exercises is extracted mostly from Chapters 3 and 4.

Handout for 24 October. Solutions to questions 1, 5, 10 to be handed in to lecturer (or via pigeon-hole) by **11.00 on 12 November**. Class in MR14 at 15.30 on Tuesday 19 November.

1. *Subadditive inequalities*. Exercises 3.1 and 3.2.
2. *Bond and site critical probabilities*. Exercise 3.3.
3. *Covering graphs*. Exercise 3.4.
4. *The problem of runs*. Exercises 3.7 and 3.8. Do not worry overmuch about the fact that you need to work with integer-values. This is ‘just noise’.
5. Exercise 3.9. You will need to know that  $\theta(\frac{1}{2}) = 0$  for bond percolation on the square lattice.
6. Exercise 3.10, suggested by a French colleague.
7. *Connective constant for  $\mathbb{A}$* . Replace each vertex  $w$  of the hexagonal lattice  $\mathbb{H}$  by an equilateral triangle, so that the hexagons become regular dodecagons (12-gons), as in the figure. Show that the resulting lattice  $\mathbb{A}$  has connective constant  $\kappa(\mathbb{A})$  which satisfies

$$\frac{1}{\kappa(\mathbb{A})^2} + \frac{1}{\kappa(\mathbb{A})^3} = \frac{1}{\kappa(\mathbb{H})}.$$



Parts of the lattices  $\mathbb{H}$  and  $\mathbb{A}$ .

8. *Positive-correlation for product measures*. Exercises 4.2 and 4.3.
9. *Russo's formula for functions*. Exercise 4.9.
10. *A critical-exponent inequality*. Exercise 5.1.
11. Sketch a solution to Exercise 5.6.