TITLES AND ABSTRACTS

Monday 5/9/22

Martin Hairer. *Stochastic quantisation of Yang-Mills.*

We report on recent progress on the problem of building a stochastic process that admits the hypothetical Yang-Mills measure as its invariant measure. One interesting feature of our construction is that it preserves gauge-covariance in the limit even though it is broken by our UV regularisation. This is based on joint work with Ajay Chandra, Ilya Chevyrev, and Hao Shen.

Terry Lyons. *Rough Paths and more scalable data science.*

The basic principle of Rough Path Theory, is that one can understand multi modal and oscillatory streams of data by considering their impact on non-linear controlled dynamical systems. The approach leads to the local description of these complex streams through signatures; connecting data science with tensor algebra. These high order signatures of the data are expensive to compute but can be re-used a huge number of times in the training of a neural net without the need to re-compute the signature from the data.

Thomas Cass. *Signatures and functions on unparameterised path space.*

The signature is a non-commutative exponential that appeared in the foundational work of K-T Chen in the 1950s. It is also a fundamental object in the theory of rough paths (Lyons, 1998). More recently, it has been proposed, and used, as part of a practical methodology to give a way of summarising multimodal, possibly irregularly sampled, time-ordered data in a way that is insensitive to its parameterisation. A key property underpinning this approach is the ability of linear functionals of the signature to approximate arbitrarily any compactly supported and continuous function on (unparameterised) path space. We present some new results on the properties of a selection of topologies on the space of unparameterised paths. Relatedly, we review some recent innovations in the theory of the signature kernel by introducing and analysing the properties of a family of so-called weighted signature kernels The talk will draw on material from two recent papers; one is joint work with William F. Turner, the other is a joint work with Terry Lyons and Xingcheng Xu.

Karen Habermann. *A polynomial expansion for Brownian motion and the associated fluctuation process.*

We start by deriving a polynomial expansion for Brownian motion expressed in terms of shifted Legendre polynomials by considering Brownian motion conditioned to have vanishing iterated time integrals of all orders. We further discuss the fluctuations for this expansion and show that they converge in finite dimensional distributions to a collection of independent zero-mean Gaussian random variables whose variances follow a scaled semicircle. We then link the asymptotic convergence rates of approximations for Brownian Lévy area which are based on the Fourier series expansion and the polynomial expansion of the Brownian bridge to these limit fluctuations. We close with a general study of the asymptotic error arising when approximating the Green's function of a Sturm-Liouville problem through a truncation of its eigenfunction expansion, both for the Green's function of a regular Sturm-Liouville problem and for the Green's function associated with the classical orthogonal polynomials.

Martin Barlow. *Stability of the elliptic Harnack Inequality.*

A manifold has the Liouville property if every bounded harmonic function is constant. A theorem of T. Lyons is that the Liouville property is not preserved under mild perturbations of the space. Stronger conditions on a space, which imply the Liouville property,are the parabolic and elliptic Harnack inequalities (PHI and EHI). In the early 1990s Grigor'yan and Saloff-Coste gave a characterisation of the parabolic Harnack inequality (PHI), which immediately gives its stability under mild perturbations. In this talk we prove the stability of the EHI. The proof uses the concept of a quasi symmetric transformation of a metric space, and the introduction of these ideas to Markov processes suggests a number of new problems. (Based on joint work with Mathav Murugan.)

Tuesday 6/9/22

Alison Etheridge. *The motion of hybrid zones (and how to stop them).*

Mathematical models play a fundamental role in theoretical population genetics and, in turn, population genetics provides a wealth of mathematical challenges. In this lecture we illustrate this by using a mathematical caricature of the evolution of genetic types in a spatially distributed population to demonstrate the role that the shape of the domain inhabited by a species can play in mediating the interplay between natural selection, spatial structure, and (if time permits) so-called random genetic drift (the randomness due to reproduction in a finite population).

Grégory Miermont. *On the enumeration of random maps with three tight boundaries and its probabilistic consequences.*

We consider the enumeration problem of graphs on surfaces, or maps, with three boundaries, also colloquially called pairs of pants. Perhaps surprisingly, a formula due to Eynard and extended by Collet-Fusy shows that this problem has a very simple and explicit solution, which becomes even simpler when one asks that the boundaries are tight, meaning that they have smallest possible length in their free homotopy class. We provide a bijective approach to this formula which consists in decomposing the graph into elementary pieces in a way that is reminiscent of certain geometric constructions of pairs of pants in hyperbolic geometry. I will also discuss the probabilistic consequences of this bijective approach by studying statistics of minimal separating loops in random maps with boundaries. (Based on joint work with Jérémie Bouttier and Emmanuel Guitter.)

Piet Lammers. *Height function delocalisation on cubic planar graphs.*

Delocalisation plays an important role in statistical physics. This talk will discuss the delocalisation transition in the context of height functions, which are integer-valued functions on the square lattice or similar two-dimensional graphs. By drawing a link with a phase coexistence result for site percolation on planar graphs, we prove delocalisation for a broad class of height functions on planar graphs of degree three. The proof also uses a new technique for symmetry breaking. The analysis includes several popular models such as the discrete Gaussian model, the solid-on-solid model, and the uniformly random K-Lipschitz function. Inclusion of the first model also implies the BKT phase transition in the XY and Villain models on the triangular lattice.

Daniel Heydecker. *Kac's process and the spatially homogeneous Boltzmann equation.*

Kac introduced a family of stochastic, many particle systems which model the behaviour of a spatially homogeneous, dilute gas, with evolution through binary elastic collisions. In the limit where the number of particles diverges, the empirical measures have the spatially homogeneous Boltzmann equation as a fluid limit. Although the Boltzmann equation itself is not explicitly probabilistic, we may use Kac's process to study the Boltzmann Equation and vice versa, and in this talk I will discuss some recent works exploring this connection.

Richard Darling. *Hidden Ancestor Graphs with Assortative Vertex Attributes.*

Synthetic vertex-labelled graphs play a valuable role indevelopment and and testing of graph machine learning algorithms. The hidden ancestor graph is a new stochastic model for a vertex-labelled multigraph G in which the observable vertices are the leaves L of a random rooted tree T, whose edges and non-leaf nodes are hidden. The likelihood of an edge in G between two vertices in L depends on the height of their lowest common ancestor in T. The label of a vertex v in L depends on a randomized label inheritance mechanism within T such that vertices with the same parent often have the same label. High label assortativity,high average local clustering, heavy tailed vertex degree distribution, and sparsity, can all coexist in this model. Subgraphs consisting of the agreement edges (end point labels agree), and the conflict edges (end point labels differ), respectively, play an important role in testing anomaly correction algorithms. Instances with a hundred million edges can be built in minutes on an average workstation with sufficient memory.

Svante Janson. *Depth-First Search in a Random Digraph.*

We study the Depth-First Search algorithm, applied to a random digraph (= directed graph). The random digraph is constructed by giving the vertices i.i.d. outdegrees, and choosing the endpoint of all arcs independently and uniformly at random. (Loops and multiple edges are allowed, but not important.) The Depth-First Search can be regarded as building a forest which eventually contains all vertices. One important property is the depth of vertices in this forest, i.e. the distance to the root. A particularly simple case is when the outdegree distribution is geometric. Its lack of memory implies that that the depths of the evrtices form a Markov process, which can be analyzed. For a general outdegree distribution, this is no longer true; as a substitute, we study a related process, which is Markov, and then are able to draw conclusions also for the depth. (Based on joint work with Philippe Jacquet.)

Wednesday 7/9/22

Jean Bertoin. *Limits of Pólya urns with innovations.*

We consider a version of the classical Pólya urn scheme which incorporates innovations. The space S of colors is an arbitrary measurable set. After each sampling of a ball in the urn, one returns C balls of the same color and additional balls of different colors given by some finite point process ξ on S. When the number of steps goes to infinity, the empirical distribution of the colors in the urn converges to the normalized intensity measure of ξ, and we analyze the fluctuations. The ratio $\rho = \mathbf{E}(C)/\mathbf{E}(R)$ of the average number of copies to the average total number of balls returned plays a key role.

Ismaël Bailleul. *The Anderson operator.*

The continuous Anderson operator H is a perturbation of the Laplace-Beltrami operator by a random space white noise potential. We consider this 'singular' operator on a two dimensional closed Riemannian manifold. One can use functional analysis arguments to construct the operator as an unbounded operator on L^2 and give almost sure spectral gap estimates under mild geometric assumptions on the Riemannian manifold. We prove a sharp Gaussian small time asymptotic for the heat kernel of H that leads amongst others to strong norm estimates for quasimodes. We introduce a new random field, called Anderson Gaussian free field, and prove that the law of its random partition function characterizes the law of the spectrum of H. We also give a simple and short construction of the polymer measure on path space and prove large deviation results for the polymer measure and its bridges. We relate the Wick square of the Anderson Gaussian free field to the occupation measure of a Poisson process of loops of polymer paths.

Vittoria Silvestri. *Planar aggregation with subcritical fluctuations and the Hastings-Levitov models.*

The ALE (Aggregate Loewner Evolution) models describe growing random clusters on the complex plane, built by iterated composition of random conformal maps. A striking feature of these models is that they can be used to define natural off-lattice analogues of several fundamental discrete models, such as Eden or Diffusion Limited Aggregation, by tuning the correlation between the defining maps appropriately. In this talk I will discuss shape theorems and fluctuations of ALE clusters, which include Hastings-Levitov clusters as particular cases, in the subcritical regime. (Based on joint work with James Norris and Amanda Turner.)

Sara Merino Aceituno. *Well-posedness and large-particle limit for a model in collective dynamics.*

The Vicsek model is a well studied model for collective dynamics where particles move at a constant speed while trying to align their orientations, up to some noise. The particle description consists of a coupled system of stochastic differential equations which in the large particle limits leads, formally, to a transport-type equation. Different modelling choices give rise to various challenges in proving the well-posedness of the equations and the large-particle derivation. (Based on joint work with Marc Briant and Antoine Diez.)

Thierry Lévy. *Schur-Weyl duality and large N limit in 2d Yang-Mills theory.*

Wilson loops are the basic observables of Yang-Mills theory, and their expectation is rigorously defined on the Euclidean plane and on a compact Riemannian surface. Focusing on the case where the structure group is the unitary group $U(N)$, I will present a formula that computes any Wilson loop expectation in almost purely combinatorial terms, thanks to the dictionary between unitary and symmetric quantities provided by the Schur-Weyl duality. This formula should be applicable to the computation of the large N limit of the Wilson loop expectations, also called the master field, and of which the existence on the sphere was proved by Antoine Dahlqvist and James Norris.