A Novel Approach to Spatially Indexed Functional Data Analysis

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Royal Statistical Society International Conference 2023
Harrogate, United Kingdom
5 September 2023

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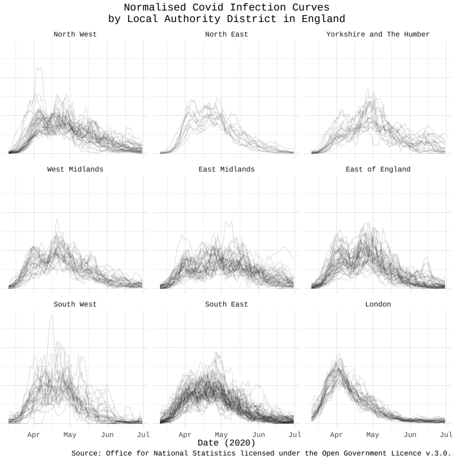
The Problem


Normalised Covid Infection Curves
by Local Authority District in England
North West
North East
Yorkshire and The Humber
West Midlands
East Midlands
East of England
South West
South East
London
Apr
May
Jun
Jul
Date (2020)
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The Model

Our model:

$$Y_u(t) = X_u(h_u^{-1}(t)),$$

perhaps with measurement error, where:

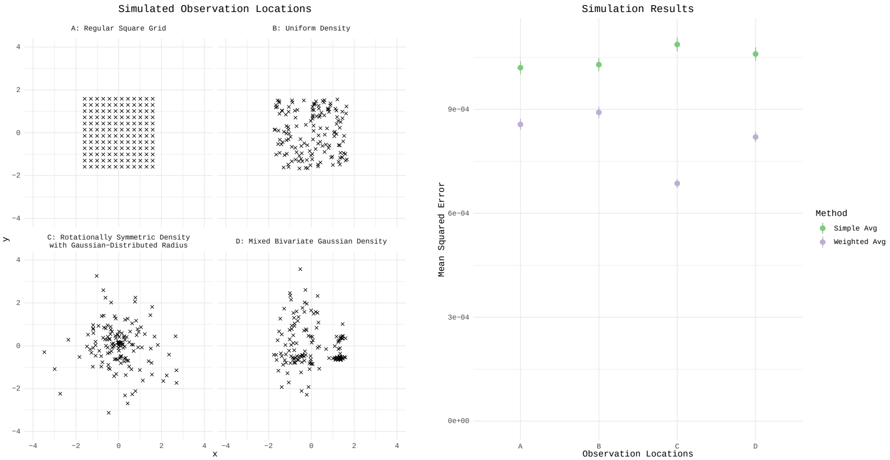
- $X_u(t) = \xi_u \mu(t) + \delta \varepsilon_u(t)$ for a random scalar field ξ, fixed mean function μ, small $\delta << \sqrt{\mathrm{Var} \xi_u}$ and random unit-norm errors $\varepsilon_u(t)$; and
- h is a random functional field on $[0,1]$ that satisfies: $h(0) = 0$, h is a diffeomorphism and $\mathbb{E} h = \mathrm{id}$.

The Method

A simplified version of our method (see the poster for more detail):

1. For each pair (i, k), we estimate the pairwise warping $g_{ik} := h_i \circ h_k^{-1}$, comparing $Y_i \circ g_{ik} \approx \xi_i \mu \circ h_k^{-1}$ to $Y_k \approx \xi_k \mu \circ h_k^{-1}$.
2. For each i, we estimate a functional variogram,
$$2\gamma_i(u_k, u_\ell) = \mathbb{E}||g_{ki} - g_{\ell i}||^2, \tag{1}$$
by making the approximation $2\gamma_i(u_k, u_\ell) \approx 2\widetilde{\gamma}_i(d(u_k, u_\ell))$.
3. We then define for $k, \ell \neq i$
$$\widehat{C}^{(i)}_{k\ell} := \widetilde{\gamma}_i(\infty) - \widetilde{\gamma}_i(d(u_k, u_\ell)), \text{ and} \tag{2}$$
$$\widehat{w}^{(i)} \propto (\widehat{C}^{(i)})^{-1} 1_{n-1}, \tag{3}$$
where 1_{n-1} is a length-$(n-1)$ vector of ones, and $\sum_k \widehat{w}^{(i)}_k = 1$.
4. We then finally estimate the h_i as a weighted mean of the $(\widehat{g}_{ki})_k$:
$$\widehat{h_i}^{-1} = \sum_{k \neq i} \widehat{w}^{(i)}_k \widehat{g}_{ki}. \tag{4}$$

Simulations



Applications